[MT-11-05-11-10]

The consideration of amplitude is independent of the considerations of phase shift and fundamental period. The student should be able to combine different amplitudes with the other two variations of the sine curve at this point.

Exercises 1.5

Give the amplitude, phase shift, and fundamental period of each of the following.

1. $y = \sin\left(x + \frac{\pi}{2}\right)$
2. $y = \cos\left(x - \frac{\pi}{2}\right)$.
3. $y = \cos(2x + \pi)$.
4. $y = \sin(3x - \pi)$.
5. $y = 2\sin(x + \pi)$.
6. $y = 3\cos(3x + 2\pi)$.
7. $y = -3\cos\left(2x - \frac{\pi}{2}\right)$.
8. $y = -2\sin\left(3x - \frac{\pi}{2}\right)$.

Graph each of the following.

9. $y = \sin\left(x + \frac{\pi}{2}\right)$.
10. $y = \cos\left(x - \frac{\pi}{2}\right)$.
11. $y = \cos(2x + \pi)$.
12. $y = \sin(3x - \pi)$.
13. $y = 2\sin(x + \pi)$.
14. $y = 3\cos(3x + 2\pi)$.
15. $y = -3\cos\left(2x - \frac{\pi}{2}\right)$.
16. $y = -2\sin\left(3x - \frac{\pi}{2}\right)$.

1.6 The other trigonometric functions

In addition to the sine function and the cosine function discussed in Section 1.2, there are four other trigonometric functions—**the tangent function, the cotangent function, the secant function,** and **the cosecant function.** These four functions are defined in terms of the sine function and the cosine function. All four are defined in this section.

The first to be considered is the tangent function, defined next.

$\tan x = \frac{\sin x}{\cos x}$

tangent function $= \{(x, y) \mid x \in R, \cos x \neq 0,$ *and* $y = \sin x/\cos x\}$.
We usually write $\tan x = y$ *to indicate that the ordered pair* (x, y) *belongs to the tangent function.*

Recall that the domain for both the sine function and the cosine function is the entire set of real numbers. This is not the case, however, for the tangent function. The tangent function is not defined at a number x for which $\cos x = 0$. If we consider the graph of the cosine function (Figure 1.28), we see that cot cos/sin –

tan sin/cos

sec 1/cos
cot 1/
csc 1/sin

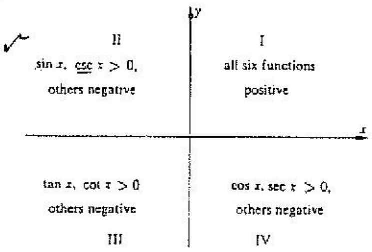


Figure 1.30

Recall from Section 1.2 that for every real number x, $\sin^2 x + \cos^2 x = 1$. If $x \neq \pi/2 + n\pi$, where $n \in J$, we can obtain

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x},$$

$$\left(\frac{\sin x}{\cos x}\right)^2 + 1 = \left(\frac{1}{\cos x}\right)^2,$$

$$\tan^2 x + 1 = \sec^2 x.$$

$\frac{\sin}{\cos}$ $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\frac{2}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$ $\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{\sqrt{3}}{3}$

Thus, if $x \neq \pi/2 + n\pi$, where $n \in J$,

$$\tan^2 x + 1 = \sec^2 x.$$

In similar fashion, if $x \neq n\pi$, where $n \in J$, we have

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x},$$

$$1 + \left(\frac{\cos x}{\sin x}\right)^2 = \left(\frac{1}{\sin x}\right)^2,$$

$$1 + \cot^2 x = \csc^2 x.$$

Thus, if $x \neq n\pi$, where $n \in J$,

$$\cot^2 x + 1 = \csc^2 x.$$

These last two equations will be of use in the homework. Since they are basic identities that will be used frequently, the student should learn them.

Exercises 1.6

1. Recalling that $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$, develop expressions for
   a. $\tan(-x)$, b. $\sec(-x)$, c. $\cot(-x)$, d. $\csc(-x)$.
2. Use the information acquired in Exercise 1 to evaluate
   a. $\tan\left(-\frac{\pi}{4}\right)$, b. $\sec\left(-\frac{\pi}{6}\right)$, c. $\csc\left(-\frac{\pi}{3}\right)$, e 5 7

d. $\cot\left(-\frac{\pi}{4}\right)$, e. $\sec\left(-\frac{\pi}{3}\right)$, f. $\tan\left(-\frac{\pi}{6}\right)$,
g. $\cot\left(-\frac{\pi}{3}\right)$, h. $\csc\left(-\frac{\pi}{4}\right)$, i. $\cot\left(-\frac{\pi}{6}\right)$,
j. $\csc\left(-\frac{\pi}{6}\right)$, k. $\tan\left(-\frac{\pi}{3}\right)$, l. $\sec\left(-\frac{\pi}{4}\right)$.

3. Using the fact that $\sin(\pi - x) = \sin x$ and $\cos(\pi - x) = -\cos x$, develop expressions for

a. $\tan(\pi - x)$, b. $\cot(\pi - x)$,
c. $\sec(\pi - x)$, d. $\csc(\pi - x)$.

4. Use the information acquired in Exercise 3 to evaluate

a. $\tan\frac{3\pi}{4}$, b. $\sec\frac{5\pi}{6}$, c. $\csc\frac{2\pi}{3}$,
d. $\cot\frac{3\pi}{4}$, e. $\sec\frac{2\pi}{3}$, f. $\tan\frac{5\pi}{6}$,
g. $\cot\frac{2\pi}{3}$, h. $\csc\frac{3\pi}{4}$, i. $\cot\frac{5\pi}{6}$,
j. $\tan\frac{2\pi}{3}$, k. $\csc\frac{5\pi}{6}$, l. $\sec\frac{3\pi}{4}$.

5. Use the fact that $\cos(x + \pi) = -\cos x$ and $\sin(x + \pi) = -\sin x$, to develop expressions for

a. $\tan(x + \pi)$, b. $\cot(x + \pi)$,
c. $\sec(x + \pi)$, d. $\csc(x + \pi)$.

6. Use the information acquired in Exercise 5 to evaluate

a. $\sec\frac{5\pi}{4}$, b. $\csc\frac{7\pi}{6}$, c. $\tan\frac{4\pi}{3}$,
d. $\cot\frac{7\pi}{6}$, e. $\csc\frac{5\pi}{4}$, f. $\cot\frac{4\pi}{3}$,
g. $\tan\frac{7\pi}{6}$, h. $\sec\frac{4\pi}{3}$, i. $\cot\frac{5\pi}{4}$,
j. $\csc\frac{4\pi}{3}$, k. $\sec\frac{7\pi}{6}$, l. $\tan\frac{5\pi}{4}$.

In each of the following exercises, use the given information to evaluate all the remaining trigonometric functions.

Example

$\cos x = \frac{15}{17}$, $\sin x < 0$.

Solution

Since $\cos x > 0$ and $\sin x < 0$, x is in the fourth quadrant. $\sec x > 0$; the others are negative.

$$\sec x = \frac{1}{\cos x} = \frac{17}{15},$$

$$\sin x = -\sqrt{1 - \cos^2 x} = -\sqrt{1 - \frac{225}{289}} = -\sqrt{\frac{64}{289}} = -\frac{8}{17},$$

$$\tan x = \frac{\sin x}{\cos x} = -\frac{8}{15},$$

$$\cot x = \frac{1}{\tan x} = -\frac{15}{8},$$

$$\csc x = \frac{1}{\sin x} = -\frac{17}{8}.$$

Example

$\tan x = -\frac{3}{4}$, $\sin x > 0$.

Solution

Since $\tan x < 0$, $\sin x > 0$, x is in the second quadrant. $\csc x > 0$; the others are negative.

$$\sec^2 x = \tan^2 x + 1 = \frac{9}{16} + 1 = \frac{25}{16},$$

$$\sec x = -\frac{5}{4}, \quad \text{since } \sec x < 0,$$

$$\cos x = -\frac{4}{5},$$

$$\cot x = -\frac{4}{3}.$$

Since $\dfrac{\sin x}{\cos x} = \tan x$,

$$\sin x = \cos x \tan x = \left(-\frac{4}{5}\right)\left(-\frac{3}{4}\right) = \frac{3}{5},$$

$$\csc x = \frac{1}{\sin x} = \frac{5}{3}$$

7. $\sec x = -\frac{13}{5}$, $\tan x > 0$.

8. $\csc x = \frac{2\sqrt{3}}{3}$, $\sec x > 0$.

9. $\tan x = \frac{1}{2}$, $\cos x > 0$.

10. $\sin x = \frac{5}{13}$, $\cot x < 0$.

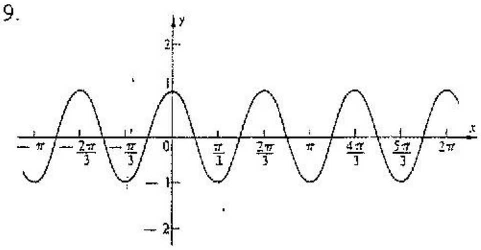
11. $\cos x = \frac{2}{\sqrt{13}}$, $\tan x < 0$.

12. $\csc x = -\frac{17}{15}$, $\cos x < 0$.

13. $\cot x = \frac{1}{3}$, $\csc x < 0$.

14. $\sec x = \frac{5}{3}$, $\sin x > 0$.

9.



Exercises 1.5

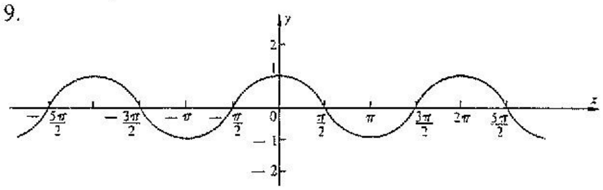
1. $1, -\frac{\pi}{2}, 2\pi$

3. $1, -\frac{\pi}{2}, \pi$

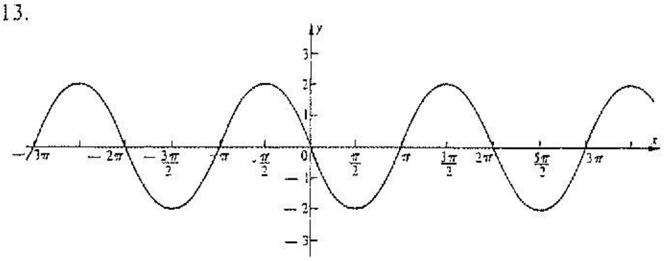
5. $2, -\pi, 2\pi$

7. $3, \frac{\pi}{4}, \pi$

9.



13.



Exercises 1.6

1. a. $\tan(-x) = -\tan x$.
   b. $\sec(-x) = \sec x$.
   c. $\cot(-x) = -\cot x$.
   d. $\csc(-x) = -\csc x$.

2. a. -1
   c. $\frac{-2}{\sqrt{3}}$ or $\frac{-2\sqrt{3}}{3}$
   e. 2
   g. $-\frac{1}{\sqrt{3}}$
   i. $-\sqrt{3}$
   k. $-\sqrt{3}$

3. a. $\tan(\pi - x) = -\tan x$.
   b. $\cot(\pi - x) = -\cot x$.
   c. $\sec(\pi - x) = -\sec x$.
   d. $\csc(\pi - x) = \csc x$.

4. a. -1
   c. $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$
   e. -2
   g. $-\frac{1}{\sqrt{3}}$

i. $-\sqrt{3}$ k. 2

5. a. $\tan(x+\pi) = \tan x$. b. $\cot(x+\pi) = \cot x$.
c. $\sec(x+\pi) = -\sec x$. d. $\csc(x+\pi) = -\csc x$.

6. a. $-\sqrt{2}$ c. $\sqrt{3}$
e. $-\sqrt{2}$ g. $\dfrac{1}{\sqrt{3}}$
i. 1 k. $-\dfrac{2}{\sqrt{3}}$ or $-\dfrac{2\sqrt{3}}{3}$

7. $\sin x = -\frac{12}{13}$, $\cos x = -\frac{5}{13}$, $\tan x = \frac{12}{5}$, $\cot x = \frac{5}{12}$, $\csc x = -\frac{13}{12}$.

9. $\sin x = \frac{1}{\sqrt{5}}$, $\cos x = \frac{2}{\sqrt{5}}$, $\cot x = 2$, $\sec x = \frac{\sqrt{5}}{2}$, $\csc x = \sqrt{5}$.

11. $\sin x = -\frac{3}{\sqrt{13}}$, $\tan x = -\frac{3}{2}$, $\cot x = -\frac{2}{3}$, $\sec x = \frac{\sqrt{13}}{2}$, $\csc x = -\frac{\sqrt{13}}{3}$

13. $\sin x = -\frac{3}{\sqrt{10}}$, $\cos x = -\frac{1}{\sqrt{10}}$, $\tan x = 3$, $\sec x = -\sqrt{10}$, $\csc x = -\frac{\sqrt{10}}{3}$

Review Exercises

1. a. $\dfrac{\sqrt{2}}{2}$ c. $\dfrac{1}{\sqrt{3}}$ e. 1 g. $\dfrac{1}{2}$ i. $\dfrac{1}{\sqrt{3}}$ k. $-\dfrac{2}{\sqrt{3}}$

12. a. $-\dfrac{\sqrt{2}}{2}$ c. 1 e. 1 g. $-\dfrac{\sqrt{3}}{2}$ i. 2

Chapter 2

Exercises 2.1

1. $\frac{\pi}{6} \in \mathscr{D}$; $\cos\frac{\pi}{6} - \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2} \neq 1$.

5. $0 \in \mathscr{D}$; $\cos(2\cdot 0) = \cos 0 = 1$; $2\cos 0 = 2(1) = 2 \neq 1$.

9. $\pi \in \mathscr{D}$; $\sin\frac{\pi}{2} = 1$; $\frac{1}{2}\sin\pi = \frac{1}{2}(0) = 0 \neq 1$.

13. $\frac{\pi}{6} \in \mathscr{D}$;
$$\frac{\tan\frac{\pi}{6} + 1}{\sin\frac{\pi}{6}} = \frac{\frac{1}{\sqrt{3}} + 1}{\frac{1}{2}}$$
$$= 2\left(\frac{1}{\sqrt{3}} + 1\right) = \frac{2}{\sqrt{3}} + 2;\ \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}} \neq \frac{2}{\sqrt{3}} + 2.$$

Exercises 2.2

The equations in exercises 9, 22, and 23 are conditional equations.

10.
$$\begin{aligned}\cos\theta\cos(-\theta) - \sin\theta\sin(-\theta) &= (\cos\theta)(\cos\theta) - (\sin\theta)(-\sin\theta)\\ &= \cos^2\theta + \sin^2\theta\\ &= 1.\end{aligned}$$