[06-01-05-MTH]

Exercises 6.2

Convert the following complex numbers from rectangular form to polar form.

1. $-1 + i$
2. $1 - \sqrt{3}\,i$
3. $-4i$
4. $3 + \sqrt{3}\,i$
5. $6 - 2\sqrt{3}\,i$
6. $4 - 4i$
7. $-1 + \sqrt{3}\,i$
8. -9

Graph each of the following complex numbers. Then convert the complex number from polar form to rectangular form.

9. $5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$
10. $3\left(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}\right)$
11. $4\left[\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right]$
12. $10\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$
13. $8\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$
14. $6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$
15. $-6\left[\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right]$
16. $4[\cos(-\pi) + i\sin(-\pi)]$

Exercises 6.3

Find the indicated products, powers, and quotients.

1. $\left[5\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right] \cdot \left[2\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right]$
2. $\left[\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right] \cdot \left[13\left(\cos(-\pi) + i\sin(-\pi)\right)\right]$
3. $\left[\sqrt{6}\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)\right] \cdot \left[\sqrt{18}\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)\right]$
4. $\left[3\left(\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right)\right] \cdot \left[7\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]$
5. $\left[3\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)\right]^3$
6. $\left[\sqrt{5}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]^6$
7. $\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]^8$
8. $\left[2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)\right]^{10}$

9. $\left[8\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right] \div \left[4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\right]$
10. $\left[35\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right] \div \left[7\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]$
11. $\left[4\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)\right] \div \left[16\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]$
12. $\left[3\left(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7}\right)\right] \div \left[\sqrt{3}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)\right]$
13. $(1 - i)^8$
14. $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^6$
15. $(3 - \sqrt{3i})^4$
16. $(-2 - 2i)^8$
17. $(1 - i)^8 \div (3 - \sqrt{3i})^4$
18. $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^6 \div (-2 - 2i)^8$

Exercises 6.2

1. $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$
3. $4\left[\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right]$
5. $4\sqrt{3}\left[\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right]$
7. $2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$
9. $\frac{5}{\sqrt{2}}+\frac{5}{\sqrt{2}}i$
11. $2-2\sqrt{3}\,i$
13. $-4\sqrt{2}-4\sqrt{2}\,i$
15. $-3\sqrt{3}+3i$

Exercises 6.3

1. $10\left(\cos\frac{13\pi}{6}+i\sin\frac{13\pi}{6}\right)$
3. $6\sqrt{3}\left(\cos\frac{14\pi}{45}+i\sin\frac{14\pi}{45}\right)$
5. $27\left(\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}\right)$
7. 1
9. $2\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$
11. $\frac{1}{4}\left[\cos\left(-\frac{2\pi}{15}\right)+i\sin\left(-\frac{2\pi}{15}\right)\right]$
13. 16
15. $144\left[\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right]$
17. $\frac{1}{9}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$

Exercises 6.4

1. $1,\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$
3. $1,\ \cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7},\ \cos\frac{4\pi}{7}+i\sin\frac{4\pi}{7},\ \cos\frac{6\pi}{7}+i\sin\frac{6\pi}{7},$
   $\cos\frac{8\pi}{7}+i\sin\frac{8\pi}{7},\ \cos\frac{10\pi}{7}+i\sin\frac{10\pi}{7},\ \cos\frac{12\pi}{7}+i\sin\frac{12\pi}{7}$
5. $\sqrt{2}\left[\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)\right],\ \sqrt{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right),$
   $\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$
7. $\sqrt[7]{5}\left(\cos\frac{\pi}{35}+i\sin\frac{\pi}{35}\right),\ \sqrt[7]{5}\left(\cos\frac{11\pi}{35}+i\sin\frac{11\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{21\pi}{35}+i\sin\frac{21\pi}{35}\right),\ \sqrt[7]{5}\left(\cos\frac{31\pi}{35}+i\sin\frac{31\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{41\pi}{35}+i\sin\frac{41\pi}{35}\right),\ \sqrt[7]{5}\left(\cos\frac{51\pi}{35}+i\sin\frac{51\pi}{35}\right),$
   $\sqrt[7]{5}\left(\cos\frac{61\pi}{35}+i\sin\frac{61\pi}{35}\right)$
9. $\sqrt{2}\left[\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)\right],\ \sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right),$
   $\sqrt{2}\left(\cos\frac{7\pi}{12}+i\sin\frac{7\pi}{12}\right),\ \sqrt{2}\left(\cos\frac{11\pi}{12}+i\sin\frac{11\pi}{12}\right),$
   $\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right),\ \sqrt{2}\left(\cos\frac{19\pi}{12}+i\sin\frac{19\pi}{12}\right)$
11. $\frac{\sqrt{3}}{2}-\frac{i}{2},\ i,\ -\frac{\sqrt{3}}{2}-\frac{i}{2}$