Answers Math 11 Trimester 2 Practice Exam 1P
Trigonometry

■ A.

[1] $\text{Cos}[\frac{-4\pi}{3}] = -\frac{1}{2}$

[2] $\text{Sin}[\frac{5\pi}{6}] = \frac{1}{2}$

[3] $\text{Cos}[\frac{\pi}{12}] = \frac{1+\sqrt{3}}{2\sqrt{2}}$

[4] $\text{Sin}[\frac{2\pi}{3}] = \frac{\sqrt{3}}{2}$

[5] $\text{Cos}[29\pi] = -1$

[6] $\text{Tan}[\frac{7\pi}{12}] = -2 - \sqrt{3}$

[7] $\text{Sin}[15^\circ] = \frac{-1+\sqrt{3}}{2\sqrt{2}}$

■ B. Answer the following. Use correct notation.

[1] Find <u>all</u> values of x for which $\sin x = \frac{-\sqrt{3}}{2}$.

$x \in \{x \ni x = \frac{4\pi}{3} + 2n\pi,\ n \in \mathbb{Z}\} \bigcup \{x \ni x = \frac{5\pi}{3} + 2n\pi,\ n \in \mathbb{Z}\}$

[2] Find <u>all</u> values of x, $0 \le x < 2\pi$, for which $\sin x \le \frac{-\sqrt{3}}{2}$.

$\frac{4\pi}{3} \le x \le \frac{5\pi}{3}$

$\{x \ni \frac{4\pi}{3} \le x \le \frac{5\pi}{3}\}$

$[\frac{4\pi}{3}, \frac{5\pi}{3}]$

[3] Find <u>all</u> values of x, $0 \le x < 2\pi$, for which $\cos x \ge \frac{-\sqrt{3}}{2}$.

$\{x \ni 0 \le x \le \frac{5\pi}{6}\} \bigcup \{x \ni \frac{7\pi}{6} \le x \le 2\pi\}$

$[0, \frac{5\pi}{6}] \bigcup [\frac{7\pi}{6}, 2\pi]$

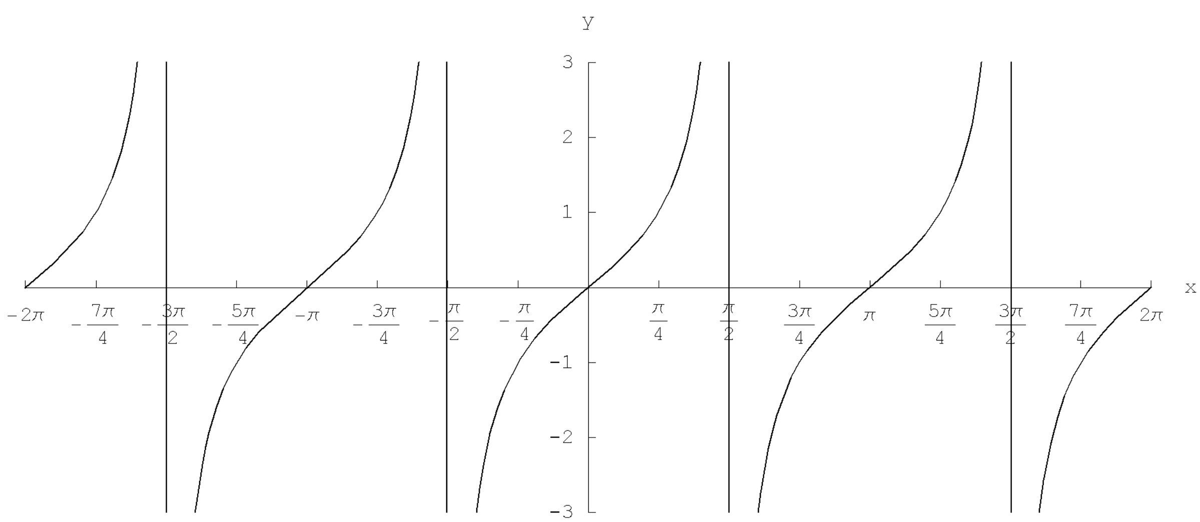
[4] Find <u>all</u> values of x, $0 \leq x < 2\pi$, for which $\tan x > 0$.

$\{x \ni 0 \leq x \leq \frac{\pi}{2}\} \bigcup \{x \ni \pi \leq x \leq \frac{3\pi}{2}\}$

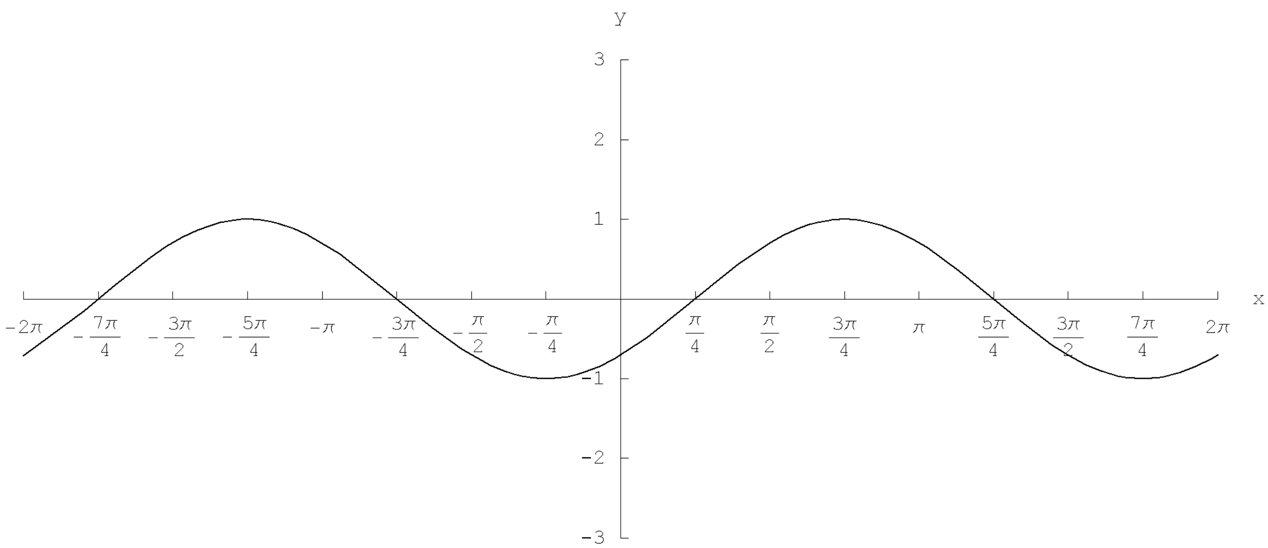
$[0, \frac{\pi}{2}] \bigcup [\pi, \frac{3\pi}{2}]$

■ C. Graph the following (neatly).

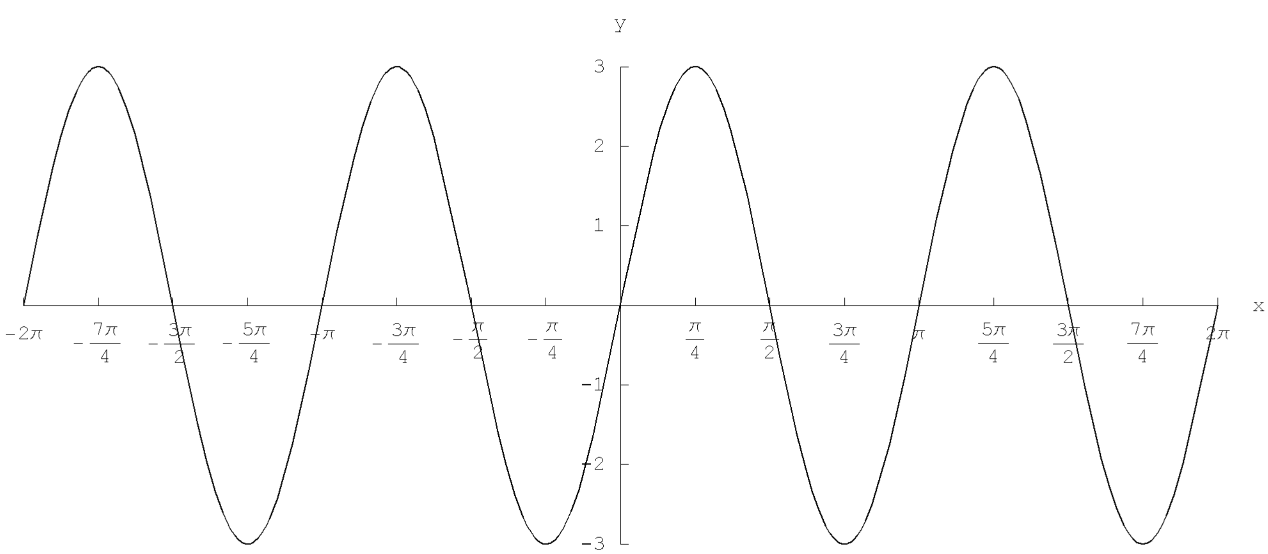
[1] The tangent function



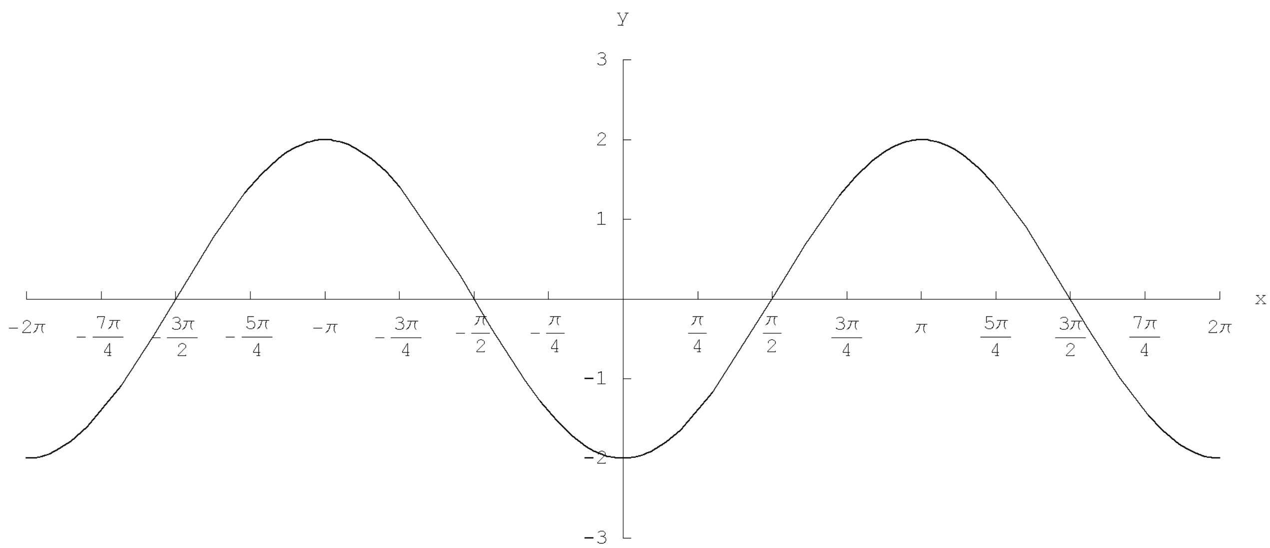
[2] $\sin(x - \frac{\pi}{4})$



[3] $3\sin 2x$

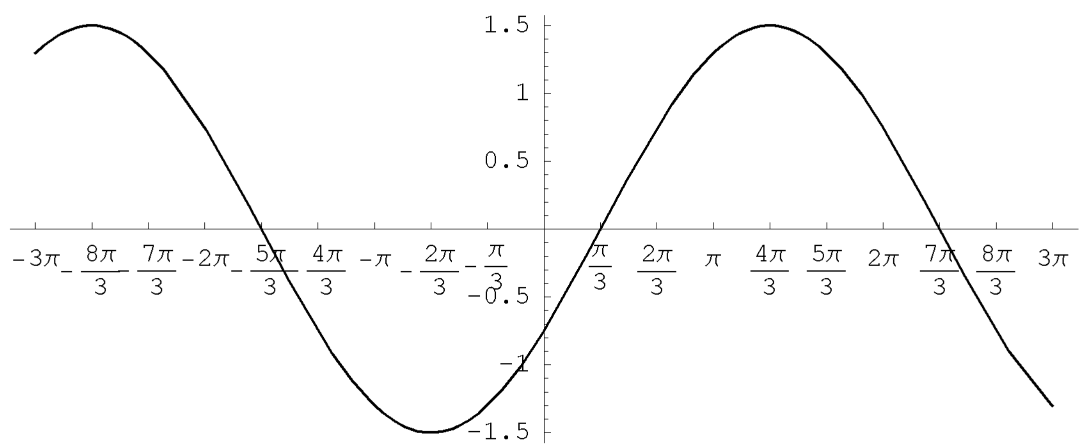


[4] $2\sin(2x - \frac{\pi}{2})$



■ **D. Write the function for each graph.**

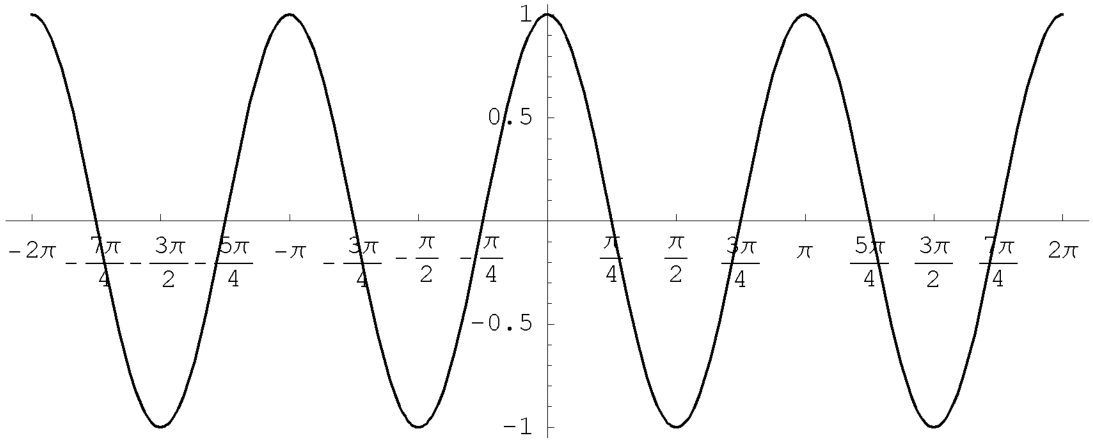
[1] The graph crosses the x-axis at $-\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}$. It has a maximum of 1.5 and a minimum of -1.5.



The function of graph [1] is:

Answer	$y = \frac{3}{2}\sin\frac{1}{2}(x - \frac{\pi}{3})$

[2] The graph crosses the x-axis at $-\frac{5\pi}{3}, \frac{3\pi}{4}, \frac{-\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$. It has a maximum of 1.0 and a minimum of -1.0.

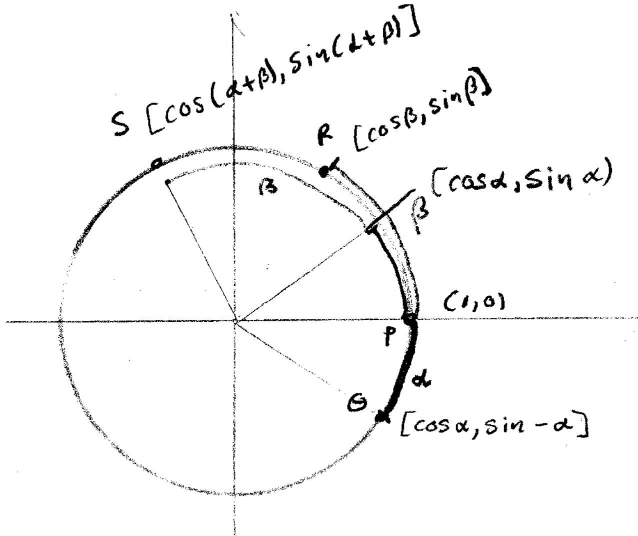


The function of graph [2] is:

Answer	$y = \sin 2(x + \frac{\pi}{4})$

■ D. Proofs

[1] Prove any one of the addition theorems that involves only sines and cosines.



$\overset{\frown}{PS} = \overset{\frown}{QR} \Longrightarrow \overline{PS} = \overline{QR} \Longrightarrow \overline{PS}^2 = \overline{QR}^2$.

$\overline{PS}^2$

$= (\cos(\alpha+\beta) - 1)^2 + (\sin(\alpha+\beta)^2 = \cos^2(\alpha+\beta) - 2\cos(\alpha+\beta) + 1 + \sin^2(\alpha+\beta)$

$= 2 - 2\cos(\alpha+\beta)$

$\overline{QR}^2$

$= (\cos\beta - \cos\alpha)^2 + (\sin\beta - \sin(-\alpha))^2$

$= (\cos\beta - \cos\alpha)^2 + (\sin\beta + \sin\alpha)^2$

$= \cos^2\beta - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\beta + 2\sin\alpha\sin\beta + \sin^2\alpha$

$= 2 - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$

$\overline{PS}^2 = \overline{QR}^2 \Longrightarrow \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

□

[2] Prove: $\boldsymbol{\sin 2\alpha = 2\sin\alpha\cos\alpha}$

$\sin(2\alpha) = \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \sin\alpha\cos\alpha = 2\sin\alpha\cos\alpha$ □

[3] Prove: $\boldsymbol{\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]}$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$ (EQ1)

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$ (EQ2)

$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$ (EQ1) + (EQ2)

$\therefore \sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha+\beta) + \sin(\alpha-\beta))$

□

[4] Prove: $\boldsymbol{\cos 2x = 2\cos^2 x - 1}$

$\cos 2x = \cos(x+x) = \cos x\cos x - \sin x\sin x = \cos^2 x - \sin^2 x$

$= \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$ □

■ E. Answer the following

[1] Find the maximum and minimum values of $\boldsymbol{y = \sin\theta + \sqrt{3}\cos\theta}$ <u>and</u> state the values of θ, $0 \le \theta < 2\pi$, at which they occur.

Let point P in the coordinate plane have coordinates $P\left(1, \sqrt{3}\right)$. Then distance origin to P is $\overline{\text{OP}} = \sqrt{1^1 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$. If $\overline{\text{OP}}$ makes angle α with positive x-axis, then

$\sin\alpha = \frac{\sqrt{3}}{2}$ and $\cos\alpha = \frac{1}{2}$.

$\sin\theta + \sqrt{3}\cos\theta = 2\left(\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta\right) = 2(\cos\alpha\sin\theta + \sin\alpha\cos\theta) = 2\sin(\alpha + \theta)$.

Now, $\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \Longrightarrow \alpha = \tan^1\sqrt{3} \Longrightarrow \alpha = \frac{\pi}{3}$.

Thus, $2\sin(\alpha + \theta) = 2\sin(\theta + \frac{\pi}{3}) = y$.

$\therefore\ y_{\text{max}} = 2$ when $\theta + \frac{\pi}{3} = 1 \Longrightarrow \theta + \frac{\pi}{3} = \frac{\pi}{2} \Longrightarrow \theta = \frac{\pi}{6}$.

$\therefore\ y_{\text{min}} = -2$ when $\theta + \frac{\pi}{3} = -1 \Longrightarrow \theta + \frac{\pi}{3} = \frac{3\pi}{2} \Longrightarrow \theta = \frac{7\pi}{6}$.

[2] Solve for θ, $0 \le \theta \le 2\pi$, if $\boldsymbol{\sin\theta = 2\sin^2\theta}$.

$2\sin^2\theta - \sin\theta = 0 \Longrightarrow \sin\theta(2\sin\theta - 1) = 0 \Longrightarrow \theta = \frac{\pi}{2}$ or

$\sin\theta = 0 \Longrightarrow \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

$2\sin\theta - 1 = 0 \Longrightarrow \sin\theta = \frac{1}{2} \Longrightarrow \theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.

[3] Solve for x, if $\boldsymbol{5\cos 3x - 1 = 4\cos 3x + 1}$

$5\cos 3x - 1 = 4\cos 3x + 1 \Longrightarrow \cos(3x) = 2 \Longrightarrow$ nosolution.

[4] Solve for x, if $\mathbf{\cos 2x - 1 = 3\cos 2x - 1}$

$2\text{Cos}[2x] = 0 \Longrightarrow \cos[2x]0 \Longrightarrow 2x = \frac{\pi}{2} \text{or} 2x = \frac{3\pi}{2}$

Thus, $S = \left\{x \ni x = \frac{\pi}{2} + 2n\pi,\ n \in \mathbb{Z}\right\} \bigcup \left\{x \ni x = \frac{3\pi}{2} + 2n\pi,\ n \in \mathbb{Z}\right\}$

[5] Solve for θ, if $\mathbf{\sin 4\theta = -\sin 3\theta}$

$\sin 4\theta = -\sin 3\theta \Longleftrightarrow \sin 4\theta = \sin\ -3\theta$

$\Longleftrightarrow 4\theta = -3\theta + 2n\pi\ \mathbf{or}\ 4\theta = \pi + 3\theta + 2n\pi$

$\Longleftrightarrow \theta = \frac{-2n\pi}{7}\ \mathbf{or}\ \theta = \pi + 2n\pi = (2n-1)\pi,\ \text{where}\ n \in \mathbb{Z}$