# Math 11 Trimester 1 Practice Exam 1P (319 Points)

*$N^{th}$ roots, functions, exponential equations*

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**Answers**

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## ■ A.

**[1] 5 [2] *a* [3] 32**

## ■ B.

**[1] 1 [2] 1 [3] $\sqrt{a}$**

## ■ C.

**[1] $Q(3, 2)$ [2] $Q(10, 9)$ [3] $Q(-3, -1)$**
**[4] (b) the y-axis.**
**[5] (d) all of the above.**

## ■ D.

**[1] $(0, \infty)$ [2] $(-\infty, \infty +)$**
**[3] $P(1, 1)$, $Q(-1, 1)$, $R(0, 0)$ You only need to say two of these.**
**[4] $P(1, 1)$, $Q(-1, -1)$, $R(0, 0)$ You only need to say two of these.**
**[5] Either $P(0, 0)$ or $Q(1, 1)$ will do.**

## ■ E.

[1]
Proof by counter-example. Let $f(x) = x + 1$ and $g(x) = 2x + 1$. Then $f(g(x)) = (2x + 1) + 1 = 2x + 2$ and $g(f(x)) = 2(x + 1) + 1 = 2x + 3$. In this case $f(g(x)) = g(f(x))$. Q.E.D.

[2]
$g(f(x)) = (2x + 1)^2 - 1 = 4x^2 + 4x + 1 - 1 = 4x^2 + 4x$.

[3]
$f(f(f(x))) = \left((x^{-1})^{-1}\right)^{-1} = \frac{1}{x}$

[4]
$f(g(x)) = \sqrt{(x + 2) + 1} = \sqrt{x + 3}$ so $f(g(1)) = 2$

[5]
$f(x) = x^5$ and $g(x) = 2x^2 + 3$

[6]
$D_f = \{x \in \mathbb{R} \ni x \geq 0\}$ and $R_f = \{x \in \mathbb{R} \ni x \geq 0\}$

## ■ F1

[1]
Solve $x = 3y - 2$ for $y$. Thus, $y = f^{-1}(x) = \frac{x-2}{3}$

[2]
Solve $x = \sqrt{y}$ for $y$. Thus, $y = f^{-1}(x) = x^2$, $x \in [0, \infty)$.

[3]
First, observe that the domain of $g$ is $\mathcal{D} = \{x \in \mathbb{R} \ni x \neq \frac{5}{2}$. Then , solve $x = \frac{1+3y}{5-2y}$ for $y$. Thus,
$y = f^{-1}(x) = \frac{5x-1}{2x+3}$.

■ **F2**

[1]
Show that $f$ is the inverse of $g$ by demonstrating that $f(g(x)) = I(x)$, where $I(x)$ is the identity element for the set of functions under the operation of composition. Thus,
$f(g(x)) = 5(\frac{x+2}{5}) - 2 = x + 2 - 2 = x = I(x)$. Q.E.D.

[2]
Show that $g$ is a 1-1 function, then note that a function is invertible iff it is a 1-1 function, hence $g$ has an inverse. To show $g$ is 1-1, show that $g(x_1) = g(x_2) \Longrightarrow x_1 = x_2$. Suppose $g(x_1) = g(x_2)$. Now, $g(x_1) = x_1^n$ and $g(x_2) = x_2^n$. Then $x_1^n = x_2^n \Longrightarrow \sqrt[n]{x_1} = \sqrt[n]{x_2} \Longrightarrow x_1 = x_2$. Since $g(x_1) = g(x_2) \Longrightarrow x_1 = x_2$, $g$ is, by definition, 1-1, therefore invertible. Q.E.D. (*Where would this proof fail if n were even instead of odd?*)

■ **G**

[1] $x = -6 \vee x = 2$ [2] $x = 6$ [3] $x = 3$