[T11-05-09-10A]

Algebra of functions

Let *f* and *g* be functions with domains A and B. Then,

$(f+g)(x) = f(x) + g(x) \qquad \mathcal{D} = A \cap B$

$(f-g)(x) = f(x) - g(x) \qquad \mathcal{D} = A \cap B$

$(fg)(x) = f(x)g(x) \qquad \mathcal{D} = A \cap B$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \qquad \mathcal{D} = \{A \cap B : g(x) \neq 0\}$

■ Notes

$(f+g)(x) = f(x) + g(x)$.

The sum $(f+g)$ on the LHS is the sum of two functions, *f* and *g*.

The sum $f(x) + g(x)$ on the RHS is the sum of two numbers, $f(x)$ and $g(x)$.

■ Example 1

For $f : f(x) = \sqrt{x}$ and $g : g(x) = \sqrt{9 - x^2}$, find $f + g$.

Solution

$(f+g) : (f+g)(x) = f(x) + g(x) = \sqrt{x} + \sqrt{9 - x^2}$, $\mathcal{D} = -3 \leq x \leq 3$.

Probably the hardest part of this is getting the domain of $f + g$. Here's how to get it in this example.- You know the square root of a number is a real number, provided that the number is non-negative. So the domain of $\sqrt{x}$ is $x \geq 0$. $\sqrt{9 - x^2}$ is a real number when $9 - x^2 \geq 0$, so $-3 \leq x \leq 3$. The largest domain on which both $\sqrt{x}$ and $\sqrt{9 - x^2}$ are real numbers is $-3 \leq x \leq 3$.