J11 Basic Analysis pages 65-85.

■ Basic formulae

■ General term of arithmetic sequence. First term a_1, common difference d.

$$a_n = a_1 + (n - 1) d$$

■ Sum of arithmetic series. First term a_1, last term a_n, common difference d.

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$S_n = \frac{n[2a_1 + (n-1)d]}{2}$$

■ General term of geometric sequence. First term a_1, common ratio r.

$$a_n = a r^{n-1}$$

■ Sum of geometric sequence. First term a_1, common ratio r.

$$r \neq 1, \qquad S_n = \frac{a(1 - r^n)}{1 - r}$$

$$r = 1, \qquad S_n = n a$$

■ Handy ideas for finding n^{th} term of a sequence.

■ Progression of differences

Given $\{a_n\}$, form $\{b_n\}$ by $b_n = a_{n+1} - a_n$, $(n = 1, 2, 3, \ldots)$. Then, $a_n = a_1 + \sum_{k=1}^{n-1} b_n$, $n \geq 2$

■ General term of $\{a_n\}$ using sum of first n terms.

Suppose a_n is the n^{th} term of $\{a_n\}$ and S_n is the sum of the first n terms of $\{a_n\}$, then $a_n = S_n - S_{n-1}$, $n \geq 2$.

■ Properties of Σ

For m,n non-negative integers such that $m \leq n$,

$$\sum_{i=m}^{n} c a_i = c \sum_{i=m}^{n} a_i$$

$$\sum_{i=m}^{n} (a_i \pm b_i) = \sum_{i=m}^{n} a_i \pm \sum_{i=m}^{n} b_i$$

■ Sum of a power

$$\sum_{i=m}^{n} c = (n - m + 1)\, c$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

■ Finding the sum of a power

Recall that to find $\sum_{i=1}^{n} i^2$ is was good to start with $(k+1)^3 - k^3 = 3k^2 + 3k + 1$. A similar strategy will work with sums of all powers. (NB: you will need explicite formulae for all powers less than the one whose sum you wish to know.)