Conditional Eqns. [3.1]

[EX 1] Solve $\cos x = \sin x$

LEFT BOARD

Soln
$$\cos x = \sin x$$
$$1 = \tan x$$

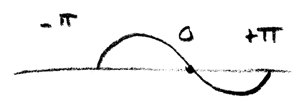
$\{\pi/4\}$ is a solution, since $\tan \pi/4 = 1$. Principal Soln

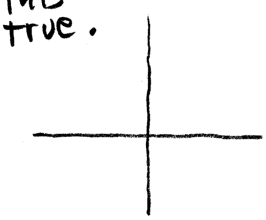
but tan is periodic, $\tan(\theta + n\pi) \equiv \tan\theta$, $n \in \mathbb{Z}$

$$\{x : x = \frac{\pi}{4} + n\pi,\ n \in \mathbb{Z}\}.$$ General Soln

Solns that lie in fundamental period are called <u>principal solutions</u>.
Complete Soln is called <u>general soln</u>.

[EX 2] $2\sin x - 3 = 6\sin x - 2$, find set S, ∋ $x \in S$ makes this true.





Soln
$$2\sin x - 3 = 6\sin x - 2$$
$$\Rightarrow \quad 4\sin x = -1$$
$$\Rightarrow \quad \sin x = -\frac{1}{4}$$
$$\Rightarrow \quad -\sin x = \frac{1}{4}$$

Now, $\sin x = \frac{1}{4} \Rightarrow x = \sin^{-1} .25 \Rightarrow x \approx .253$

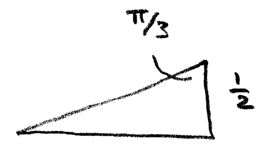
* Since $-\sin\theta = \sin(-\theta)$, $x = -0.253$

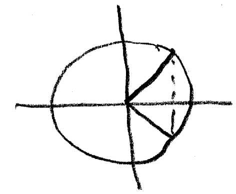
* Since $\sin(\pi + \theta) = -\sin\theta$, $x = \pi + 0.253$

Use periodicity to extend these:

General Solution
$$\therefore\ S = \{x \ni x = -0.253 + 2n\pi,\ n \in \mathbb{Z}\} \cup \{x \ni x = \pi + .253 + 2n\pi,\ n \in \mathbb{Z}\}$$

[EX 3] $2 + \cos\theta = 1 + 3\cos\theta$





$\Rightarrow \quad 2\cos\theta = 1$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

$\Rightarrow \quad \cos\,\theta = \frac{\pi}{3}$ or $\cos\,\theta = -\frac{\pi}{3}$

$\therefore \quad S = \{x \ni x = \frac{\pi}{3} + 2n\pi,\ n \in \mathbb{Z}\} \cup \{x \ni x = -\frac{\pi}{3} + 2n\pi,\ n \in \mathbb{Z}\}$

[EX 4] $\sin^2 x + 7\sin x + 6 = 0$

$\Rightarrow \quad (\sin x + 6)(\sin x + 1) = 0$

$\Rightarrow \quad \sin x = -6$ OR $\sin x = -1$

$\emptyset$ $\qquad \Rightarrow \quad x = \frac{3\pi}{2}$

$S = \{x \ni x = \frac{3\pi}{2} + 2n\pi,\ n \in \mathbb{Z}\}$

Above are easy.

- one function
- same period, 2π for sin, cos; π for tan.

More C-E [3.2]

Harder: two functions
: Periods other than 2π or π for Tan.
: will use more identities

[EX 1] Solve $2\sin 4x = 1$

Soln. Let $4x = \theta$ (substitution)

then $2\sin\theta = 1$

$$\sin\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad \theta = \frac{5\pi}{6} + 2n\pi, \ n \in \mathbb{Z}$$

Since $\theta = 4x$,

$$4x = \frac{\pi}{6} + 2n\pi \quad \text{or} \quad 4x = \frac{5\pi}{6} + 2n\pi, \ n \in \mathbb{Z}$$

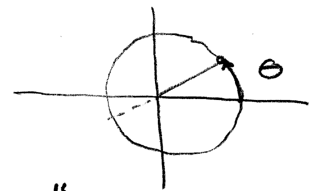
$$x = \frac{\pi}{24} + \frac{n\pi}{2} \quad \text{OR} \quad x = \frac{5\pi}{24} + \frac{n\pi}{2}$$

$$\therefore \ S = \left\{ x \ni x = \frac{\pi}{24} + \frac{n\pi}{2}, n \in \mathbb{Z} \right\} \cup \left\{ x \ni x = \frac{5\pi}{24} + \frac{n\pi}{2}, n \in \mathbb{Z} \right\}$$

[EX 2] $\cos \frac{y}{3} + 2 = 0$

Soln

Let $\theta = \frac{y}{3}$. Then,



$3\cos\theta + 2 = 0$

$\Rightarrow \quad \cos\theta = -\frac{2}{3}$

$\Rightarrow \quad -\cos\theta = \frac{2}{3}$ ← NOTE: This is "minus cos θ" NOT "cos minus θ"

$\cos\theta = \frac{2}{3} \Rightarrow \theta = 0.841 \quad \because \quad \cos^{-1}\frac{2}{3} = 0.841$

uses
$-\cos\theta = \cos(\pi - \theta) = \cos(\pi + \theta)$

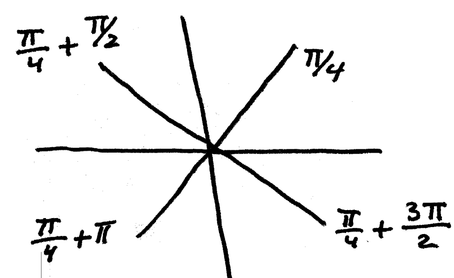
So, $\theta = \pi - 0.841 + 2n\pi$ or $\theta = \pi + 0.841 + 2n\pi$

Since $y/3 = \theta$,

$\therefore \; S = \{y : y = 3\theta - 2.523 + 6n\pi,\ n \in \mathbb{Z}\} \cup \{y : y = 3\pi + 2.523 + 6n\pi,\ n \in \mathbb{Z}\}$

[EX 4] $\sec^2\alpha + \tan^2\alpha = 3$. [2 functions. Rewrite as one function]

Soln



$\sec^2\alpha + \tan^2\alpha = 3$

$\Rightarrow \sec^2\alpha + (\sec^2\alpha - 1) = 3$

$\Rightarrow 2\sec^2\alpha = 3$

$\Rightarrow \sec^2\alpha = 2$

$\Rightarrow \sec\alpha = \sqrt{2}$ or $\sec\alpha = -\sqrt{2}$

$\therefore \; \alpha = \frac{\pi}{4} + 2n\pi$ or $\alpha = -\pi/4 + 2n\pi$, $n \in \mathbb{Z}$

or OR

$\alpha = \frac{3\pi}{4} + 2n\pi$ $\quad \alpha = \frac{5\pi}{4} + 2n\pi$

$\therefore \; S = \{x : x = \frac{\pi}{4} + \frac{n\pi}{2},\ n \in \mathbb{Z}\}$

HARD : cannot solve $x \sin x - 1 = 0$

: two funcions with different fundamental periods

[EX-1] $\sin 3x = \sin 7x$ "one" function, different periods

☆ <u>FACT</u>: $\sin x = \sin y$ iff (i) for some integer n, $y = x + 2n\pi$
or (ii) for some integer n, $y = (\pi - x) + 2n\pi$

Recall $\sin(\pi - x) = \sin x$

<u>Soln</u>: $\sin 3x = \sin 7x$

If

$7x = 3x + 2n\pi, \ n \in \mathbb{Z}$ OR $7x = (\pi - 3x) + 2n\pi \ , n \in \mathbb{Z}$.

$4x = 2n\pi$
$x = \frac{n\pi}{2}$

$10x = \pi + 2n\pi$
$x = \frac{\pi}{10} + 5n\pi$

$\therefore \ S = \{x \mid x = \frac{n\pi}{2}, n \in \mathbb{Z}\} \cup \{x \mid x = \frac{\pi}{10} + 5n\pi, n \in \mathbb{Z}\}$

[EX 2] $\cos 5x = -\cos 2x$

$\Rightarrow \cos 2x = \cos(\pi - 5x)$ OR $\cos 2x = \cos(\pi + 5x)$

$\because -\cos x = \cos(\pi - x) = \cos(\pi + x)$

$\Rightarrow 2x = (\pi - 5x) + 2n\pi$, $n \in \mathbb{Z}$

$7x = \pi + 2n\pi$

$x = \frac{\pi}{7} + \frac{2n\pi}{7}$

$2x = (\pi + 5x) + 2n\pi$ BY FACT Above

$-3x = \pi + 2n\pi$

$x = -\frac{\pi}{3} - \frac{2n\pi}{3}$

$\because \cos x = \cos(-x)$

$\therefore S = \{x \mid x = \frac{\pi}{7} + 2n\pi, n \in \mathbb{Z}\} \cup \{x \mid x = -\frac{\pi}{3} + \frac{2n\pi}{3}, n \in \mathbb{Z}\}$

[EX 3] $\sin 2\theta = \cos 3\theta$

Soln

$\Rightarrow \sin 2\theta = \sin(\frac{\pi}{2} - 3\theta)$ $\because \cos 3\theta = \sin(\frac{\pi}{2} - 3\theta)$

$\Rightarrow \frac{\pi}{2} - 3\theta = 2\theta + 2n\pi$ OR $\frac{\pi}{2} - 3\theta = \pi - 2\theta + 2n\pi$ {FACT ABOVE $n \in \mathbb{Z}$

$-5\theta = -\frac{\pi}{2} + 2n\pi$

$\theta = \frac{\pi}{10} - \frac{2n\pi}{5}$ or $\theta = -\frac{\pi}{2} - 2n\pi$

$\theta = \frac{\pi}{10} + \frac{2n\pi}{5}$ OR $\theta = -\frac{\pi}{2} + 2n\pi$

$\therefore S = \{\theta \ni \theta = \frac{\pi}{2} + 2n\pi, n \in \mathbb{Z}\} \cup \{\theta \ni \theta = -\frac{\pi}{2} + 2n\pi\}, n \in \mathbb{Z}\}$