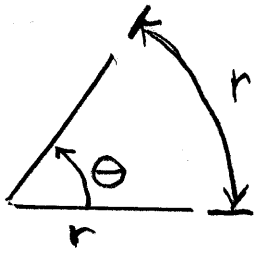
"If ever there was a reason to keep a special summary notebook section, it is now"

J11 pps 30 – 34



## Radian Measure

**Def:** The measure of an angle at the center of a circle that is subtended by an arc of the circle one radius long is said to be one <u>radian</u>.



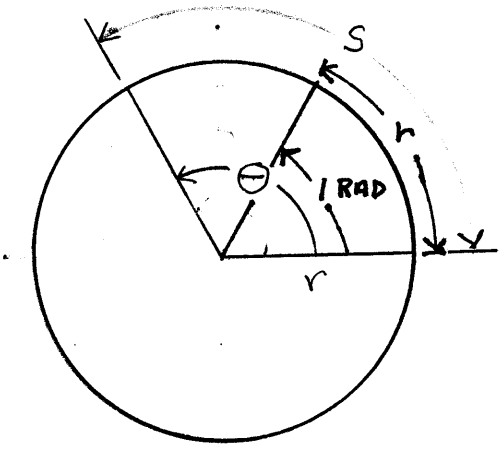

$\theta$ = 1 radian

More generally, the number of radians in an angle that is subtended by an arc s units long is

$$\theta = \frac{s}{r} . \qquad (1)$$

<u>NB</u>: (1) is true only for angles in radians.

most common form $\boxed{s = r\theta}$




What's the radian measure of $\theta$?

① – Ans. However many r's s is, that is the radian measure of $\theta$.

OR

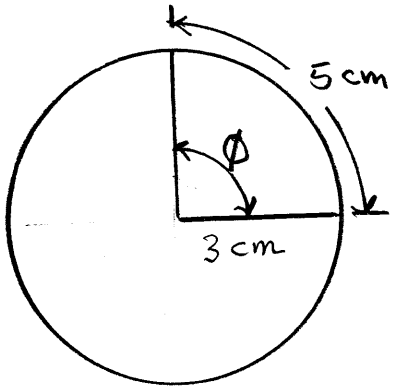
② – Ans: $s \div r$ is how many radii s is and $s \div r$ is the size of $\theta$ in radians.

→

## Notes about radian measure.

1. A radian is dimensionless, because it is a ratio.

$$\phi = \frac{s}{r} = \frac{5\,\text{cm}}{3\,\text{cm}} = \frac{5}{3} \text{ no units.}$$




2. The degree is an arbitrary unit, the radian is a natural unit.

## How many radians in a circle?

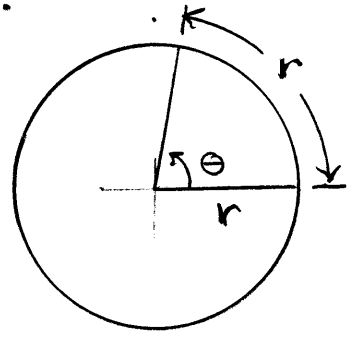
$$\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$$

$2\pi r$ is Circumference of Circle radius r.

## What is relationship of degree to radian?

$$\frac{r}{2\pi r} = \frac{\theta}{360^\circ} \quad \because \theta = 1\,\text{rad},\ s = r$$

$$\Rightarrow \quad \theta = \frac{180^\circ}{\pi} \approx 57.2958^\circ$$




$s = r$ when $\theta = 1$ radian

So,

$$1 \text{ radian} = \frac{180^\circ}{\pi}$$

and

$$1^\circ = \frac{\pi}{180} \text{ radians.}$$

(2)

[EX 1]

$120° \longrightarrow$ ? radians

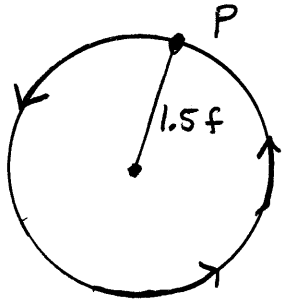
$$\overset{2}{\cancel{120}}\left(\frac{\pi}{\cancel{180}_{3}}\right) = \frac{2}{3}\pi \text{ radians.}$$

* we usually do not say or write "radian" so ans. is just $\frac{2}{3}\pi$.

[EX 2] $\frac{\pi}{6} \longrightarrow$ ? deg.

$$\frac{\pi}{6}\left(\frac{180}{\pi}\right) = \frac{180}{6} = 30°$$

[EX 3] A wheel is rotating at rate of 4 rad $s^{-1}$. Find the distance traveled in 1 sec by a point P on the wheel 1.5 ft from center.




rotation rate

4 rad $s^{-1}$

Soln:

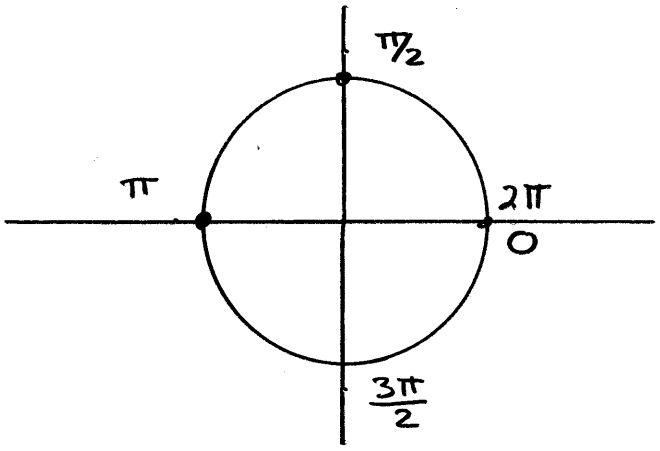
In 1 sec, wheel turns through 4 radians.

$\theta$ is changing at rate of 4 rad/s.

$$s = r\theta$$
$$= (1.5\text{ f})(4)$$
$$= 6\text{ f}$$

$\therefore$ P travels a distance of 6 feet

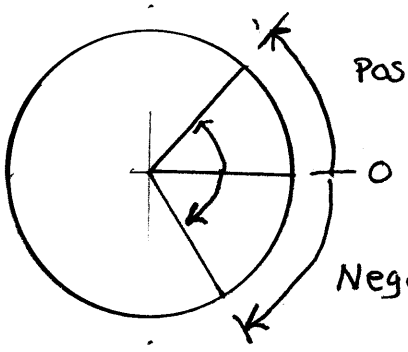
→



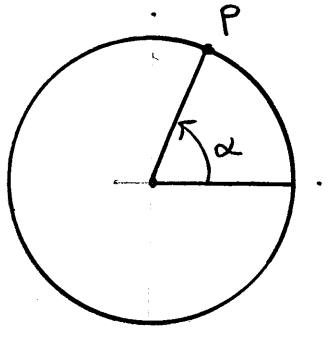

$180^\circ = \pi$, $90^\circ = \frac{\pi}{2}$, $270^\circ = \frac{3\pi}{2}$, $360^\circ = 2\pi$

* "Circular measure" means "radian measure"

: Positive / negative




: General angle




$\alpha + 360^\circ$ ends up at P
$\alpha + 720^\circ$ ends up at P
$\alpha - 360^\circ$ " "
$\alpha - 720^\circ$ " "

* $\alpha + 360n$, $n \in \mathbb{Z}$ ends up at P

≡ $\alpha + n\pi$, $n \in \mathbb{Z}$, ends up at P.

ASSIGN

PPS 30-32 #1, 2, 5
PPS 32-34 #6, 7, 9

OR

[P2-11-05-10-12] [A] #1, 2

END