P67

[1.1] $-2, -5, -8, -11, -14$

[1.2] $1, 1, 3, 7, 13$

[1.3] $-1, 1, -1, 1, -1$

P68

[1.1] $a_{20} = 4 + 19 \cdot 3 = 61$

[1.2] $a_{25} = 7 + 24(-\frac{1}{3}) = -1$

[2.1] $a = 23 \quad d = 7 \Rightarrow a_n = 23 + (n-1)7$

[2.2] $a = 2 \quad d = -\frac{3}{4} \Rightarrow a_n = 2 + (n-1)(-\frac{3}{4})$

[3.1] $a_n = 8 + (n-1)(-3) = -37$

$-3(n-1) = -45$

$n - 1 = 15$

$n = 16$

$\therefore$ The 16^{th} term

[4] $\begin{bmatrix}(3-1)d + a = -4\\(10-1)d + a = 38\end{bmatrix} = \begin{bmatrix}2d + a = -4\\9d + a = 38\end{bmatrix} \Rightarrow \begin{matrix}d = 6\\a = -16\end{matrix}$

$\therefore \quad a_n = -16 + 6(n-1)$

[5] $a = 3, a_{15} = 94$

$94 = 3 + (15-1)d \Rightarrow d = \frac{91}{14}$

$a_5 = 3 + 4(\frac{91}{14}) = 29$

$a_{10} = 3 + 9(\frac{91}{14}) = 61.5$

[6] $\forall\ p, q \in \mathbb{R}$ constant, $n \in \mathbb{Z}^+$.

$a_n = pn+q$ and $a_{n-1} = p(n-1)+q$

then $a_n - a_{n-1} = pn+q - pn + p - q = p.$

$\therefore$ $\{pn+q\}$ is an arith progression, $d = p.$

[7] $\forall$ $\{a_n\}, \{b_n\}$ arith. sequence.

Then $a_n = a + (n-1)d_a$ and $b_n = b + (n-1)d_b.$

$$\{a_n + b_n\} = (a + nd_a - d_a) + (b + nd_b - d_b)$$

$$= (a+b) + d_a(n-1) + d_b(n-1)$$

$$= (a+b) + (d_a + d_b)(n-1)$$

$$\{a_n + b_n\} = c + (n-1)d_c, \quad \begin{cases} a+b=c \\ d_a + d_b = d_c \end{cases}$$

[8.1] $S_{10} = \frac{10}{2}[7+61] = \frac{10}{2} \cdot 68 = 340$

[8.2] $S_{13} = \frac{13}{2}[-20+(12)(4)] = \frac{13}{2} \cdot 28 = 13 \cdot 14 = 182$

[8.3] $S_n = \frac{n}{2}[21-117]$

$-117 = 21+(n-1)(-6) = 21-6n+6 = 27-6n$

$\Rightarrow \quad 6n = +117+27$

$= 144$

$n = \frac{144}{6} = 24$

$S_n = \frac{24}{2}[21-117] = -1152$

[9.1] use $S_n = \frac{n}{2}(a+a_n)$ with $a_n = n$.

LHS $= 1+2+3+\ldots+n = \frac{n}{2}(1+n) =$ RHS

[9.2] use $S_n = \frac{n}{2}(a+a_n)$ with $a_n = 2n-1$

LHS $= 1+3+5+\ldots+(2n-1)$

$= \frac{n(2n-1+1)}{2}$

$= \frac{2n^2}{2}$

$= n^2$

$=$ RHS

[10.1] $a = 1,\ a_n = 100,\ n = 100$

$$S_{100} = \frac{100}{2}(1+100) = 5050$$

[10.2]
$$\begin{aligned} S &= 3 + 6 + 9 + 12 + \ldots + 192 + 195 + 198 \\ &= 3\cdot1 + 3\cdot2 + 3\cdot3 + 3\cdot4 + \ldots + 3\cdot64 + 3\cdot65 + 3\cdot66 \\ &= 3(1+2+3+\ldots+66) \\ &= 3\left[\frac{66}{2}[1+66]\right] \\ &= \frac{1}{2}\cdot 3\cdot 66\cdot 67 \\ &= 6633 \end{aligned}$$

[11] $S_n = 297$, $a = 45$, $d = -3$, $n = ?$ } arithmetic seq.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$297 = \frac{n}{2}[90 + (n-1)(-3)]$$

$$= \frac{n}{2}[90 - 3n + 3]$$

$$= \frac{n}{2}[93 - 3n]$$

$$= \frac{93n}{2} - \frac{3}{2}n^2$$

$$\Rightarrow \quad 594 = 93n - 3n^2$$

$$\Rightarrow \quad 3n^2 - 93n + 594 = 0$$

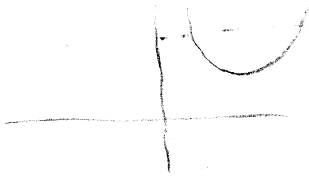
$$\Rightarrow \quad n^2 - 31n + 198 = 0$$

$$\Rightarrow \quad (n-22)(n-9) = 0$$

$$\Rightarrow \quad n = 22 \text{ OR } n = 9$$

$$\therefore \quad n = 22 \text{ or } n = 9$$

$3[90-15] = 225$



$198 = 2 \cdot 3 \cdot 3 \cdot 11$

$n = 22$

$\frac{22}{2}[90 + (21)(-3)]$

$= 11[27]$

$= 297$ ✓

$n = 9$

$\frac{9}{2}[90 + (8)(-3)]$

$= \frac{9}{2}[66]$

$= 297$ ✓