The problem set asks you to use mathematical induction. It does not claim that only that technique will work. Nor does it claim that induction is even the best technique. The problems are there for you to use to get practice with the technique of Mathematical induction.

In what follows, I will do just the inductive step of each proof. But, you must do the entire proof.

[7] suppose S_k is true. I.e.

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$

[redacted]

show S_{k+1} true. I.E

$$\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$$

$$LHS = \sum_{i=1}^{k+1} i$$

$$= \sum_{i=1}^{n} i + (k+1)$$

$$= \frac{k(k+1)}{2} + (k+1) \qquad \text{USED hypothesis}$$

$$= (k+1)\left(\frac{k}{2} + 1\right)$$

$$= (k+1)\left(\frac{k+2}{2}\right)$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= RHS$$

[8] spose $\sum_{i=1}^{k} 2i = k^2 + k$

show $\sum_{i=1}^{k+1} 2i = (k+1)^2 + (k+1)$

$n^2 + 2n + 1 + n + 1$

$n^2 + 3n + 2$

$$LHS = \sum_{i=1}^{k+1} 2i$$

$$= \sum_{i=1}^{k} 2i + 2(k+1)$$

$$= k^2 + k + 2(k+1)$$ USED HYP

$$= k^2 + 3k + 2$$

$$= k^2 + 2k + 1 + k + 1$$

← I'm not as clever as this suggests. I know n^2+3n+2 should equal $(n+1)^2 + (n+1)$; so that guides me to $n^2+2n+1+n+1$

$$= (k+1)^2 + (k+1)$$

$$= RHS$$

□

[9] Spose S_k: $\sum_{i=1}^{k} (2i-1) = k^2$

show S_{k+1}: $\sum_{i=1}^{k+1} (2i-1) = (k+1)^2$

$$
\begin{aligned}
LHS &= \sum_{i=1}^{k+1} (2i-1) \\
&= \sum_{i=1}^{k} (2i-1) + 2(k+1)-1 \\
&= k^2 + 2k + 1 && \text{USED HYP} \\
&= (k+1)^2 \\
&= RHS
\end{aligned}
$$

$\square$

[10] Spose S_j: $\sum_{k=1}^{j} (1+2k) = j^2 + 2j$

show S_{j+1}: $\sum_{k=1}^{j+1} (1+2k) = (j+1)^2 + 2(j+1)$

note: I write S_j instead of S_k since I want to avoid $\underline{k}$ which is used as dummy variable of the Σ.

$$
\begin{aligned}
LHS &= \sum_{k=1}^{j+1} (1+2k) \\
&= \sum_{k=1}^{j} (1+2k) + 1 + 2(j+1) \\
&= j^2 + 2j + 1 + 2(j+1) && \text{USED HYP} \\
&= (j+1)^2 + 2(j+1) \\
&= RHS
\end{aligned}
$$

$\square$

[11] S_j: $\displaystyle\sum_{k=1}^{j} (5+4k) = 2j^2 + 7j$

S_{j+1}: $\displaystyle\sum_{k=1}^{j+1} (5+4k) = 2(j+1)^2 + 7(j+1)$

$$LHS = \sum_{k=1}^{j+1} (5+4k)$$

$$= \sum_{k=1}^{n} (5+4k) + 5 + 4(j+1)$$

$$= 2j^2 + 7j + 5 + 4(j+1) \qquad \text{used HYP}$$

$$= 2j^2 + 7j + 5 + 4j + 4$$

$$= 2j^2 + 4j + 2 + 7j + 7$$

$$= 2(j+1)^2 + 7(j+1)$$

$\square$

[R] $S_j: \sum_{k=1}^{j}[a+(k-1)d] = \frac{j}{2}[2a+(j-1)d]$

$S_{j+1}: \sum_{k=1}^{j+1}[a+(k-1)d = \frac{j+1}{2}[2a+(j+1-1)d]$

$$LHS = \sum_{k=1}^{j+1}[a+(k-1)d]$$

$$= \sum_{k=1}^{j}[a+(k-1)d] + [a+(j+1-1)d] \quad \text{use HYP}$$

$$= \frac{j}{2}[2a+(j-1)d] + [a+jd]$$

$$= \; = aj + \left(\frac{j^2-j}{2}\right)d + a + jd$$

$$= [2aj + (j^2-j)d + 2a + 2dj]\frac{1}{2}$$

$$= [2aj + j^2 d - jd + 2a + 2dj]\frac{1}{2}$$

$$= [dj^2 + dj + 2a + 2aj]\frac{1}{2}$$

$$= [dj^2 + dj + 2a(1+j)]\frac{1}{2}$$

$$= [dj(j+1) + 2a(j+1)]\frac{1}{2}$$

$$= \frac{j+1}{2}[dj + 2a]$$

$$= \frac{j+1}{2}[2a + (j+1-1)d]$$

$$= RHS$$

$\square$