# [06-02-21-T11] Answers

■ **The following are the inductive steps to the proofs. The student should write the entire proof in correct form.**

[13]

Spose $S_k : \sum_{i=1}^{k} 2^{i-1} = 2^k - 1$

Show $S_{k+1} : \sum_{i=1}^{k+1} 2^{i-1} = 2^{k+1} - 1$

LHS $= \sum_{i=1}^{k+1} 2^{i-1}$

$= \sum_{i=1}^{k} 2^{i-1} + 2^{k+1-1}$

$= 2^k - 1 + 2^k$ HYP

$= 2 \cdot 2^k - 1$

$= 2^{k+1} - 1$

$=$ RHS

□

[14]

Spose $S_k : \sum_{i=1}^{k} (\frac{1}{2})^i = 1 - (\frac{1}{2})^k$

Show $S_{k+1} : \sum_{i=1}^{k+1} (\frac{1}{2})^i = 1 - (\frac{1}{2})^{k+1}$

$\text{LHS} = \sum_{i=1}^{k+1} (\frac{1}{2})^i$

$= \sum_{i=1}^{k} (\frac{1}{2})^i + (\frac{1}{2})^{k+1}$

$= 1 - (\frac{1}{2})^k + (\frac{1}{2})^{k+1}$ HYP

$= 1 - (\frac{1}{2})^k + (\frac{1}{2})^k \cdot \frac{1}{2}$

$= 1 - (\frac{1}{2})^k (1 - \frac{1}{2})$

$= 1 - \frac{1}{2} (\frac{1}{2})^k$

$= 1 - (\frac{1}{2})^{k+1}$

$= \text{RHS}$

□

[15]

Spose $S_j : \sum_{k=1}^{j} a \cdot r^{k-1} = \frac{a(r^j - 1)}{r-1}, r \neq 1$

Show $S_{j+1} : \sum_{k=1}^{j+1} a \cdot r^{k-1} = \frac{a(r^{j+1} - 1)}{r-1}, r \neq 1$

$\text{LHS} = \sum_{k=1}^{j+1} a \cdot r^{k-1}$

$= \sum_{k=1}^{j} a \cdot r^{k-1} + a \cdot r^{j+1-1}$

$= \frac{a(r^j - 1)}{r-1} + a \cdot r^j$ HYP

$= \frac{ar^j - a + (ar^j)(r-1)}{r-1}$

$= \frac{ar^j(1 + r - 1) - a}{r-1}$

$= \frac{ar^{j+1} - a}{r-1}$

$= \frac{a(r^{j+1} - 1)}{r-1}$

$= \text{RHS}$

□

Null