[P1-05-11-02]

It will be seen from the exercises just completed that

An expression of the form $x^2 + kx$ may be changed to a perfect square by adding the square of one half of k, the coefficient of x.

Exercises [A]

Copy and fill in the blanks.

1. $a^2 + 2a + \ ? \ = (\ \ ?\ \ )^2$
2. $b^2 - 2b + \ ? \ = (\ \ ?\ \ )^2$
3. $c^2 + \ \ c + \ ? \ = (\ \ ?\ \ )^2$
4. $d^2 - \ \ d + \ ? \ = (\ \ ?\ \ )^2$
5. $m^2 + 3m + \ ? \ = (\ \ ?\ \ )^2$
6. $n^2 - 3n + \ ? \ = (\ \ ?\ \ )^2$
7. $p^2 + \frac{2}{3}p + \ ? \ = (\ \ ?\ \ )^2$
8. $x^2 - 7x + \ ? \ = (\ \ ?\ \ )^2$
9. $y^2 + \frac{3}{4}y + \ ? \ = (\ \ ?\ \ )^2$
10. $w^2 - \frac{1}{2}w + \ ? \ = (\ \ ?\ \ )^2$

Solve, leaving irrational answers in simplest radical form:

11. $x^2 - 5 = 0$
12. $x^2 - 5x = 0$
13. $x^2 - 5x - 6 = 0$
14. $(x - \frac{5}{2})^2 = 16$
15. $(x - \frac{5}{2})^2 = 17$
16. $(x - 2)^2 + (x - 3)^2 = 13$
17. $r^2 = 4r$
18. $r^2 = 4$
19. $\dfrac{x - 9}{3} = \dfrac{4}{x - 9}$
20. $\dfrac{12}{x(x - 9)} = 1 - \dfrac{9}{x}$
21. $(a - 3)(a - 1) + 1 = 2a - 1$
22. $(a - 3)(a - 1) = 2a - 1$

Example 1. Solve the equation $x^2 + 8x = 2$.

Solution: $x^2 + 8x = 2$

Add 16 (the square of one half of 8) to both sides.

$$x^2 + 8x + 16 = 2 + 16$$
$$(x + 4)^2 = 18$$
$$x + 4 = \sqrt{18} \quad \text{or} \quad x + 4 = -\sqrt{18}$$
$$x = -4 + 3\sqrt{2} \quad | \quad x = -4 - 3\sqrt{2}$$

Check:

If $x = -4 + 3\sqrt{2}$,

$$\begin{array}{l|l} (-4 + 3\sqrt{2})^2 + 8(-4 + 3\sqrt{2}) & 2 \\ 16 - 24\sqrt{2} + 18 - 32 + 24\sqrt{2} & \\ 2 & 2 \end{array}$$

If $x = -4 - 3\sqrt{2}$,

$$\begin{array}{l|l} (-4 - 3\sqrt{2})^2 + 8(-4 - 3\sqrt{2}) & 2 \\ 16 + 24\sqrt{2} + 18 - 32 - 24\sqrt{2} & \\ 2 & 2 \end{array}$$

Exercises [A-1]

Solve by completing the square, leaving irrational answers in simplest radical form:

1. $x^2 - 4x = 5$
2. $x^2 - 12x = -4$
3. $6s = s^2 - 17$
4. $x^2 + 3x = 2$
5. $p^2 - 2 = 7p$
6. $x = x^2 - 3$

Carry out the indicated operations, writing results to the nearest hundredth:

7. $5 - 2.646$
8. $-2 + 5.975$
9. $3 + 8.614$
10. $-7 - 4.930$
11. 6 ± 5.701
12. -3.4 ± 6.132
13. $\dfrac{-5 \pm 2.881}{2}$

Solve, giving irrational answers to the nearest hundredth:

14. $y^2 = 6y - 3$
15. $b^2 = 8b - 4$
16. $y^2 = 1.2y + .8$
17. $a^2 = 2.4a - 1$
18. $y^2 = y + 6$
19. $x^2 + x - 1 = 0$

Exercises [A-2]

Solve by completing the square, leaving irrational answers in simplest radical form:

1. $y^2 + 8y = 2$
2. $z^2 + 10z = 24$
3. $a^2 = 6 - 10a$
4. $y^2 = 5y - 2$
5. $z^2 = 11z + 4$
6. $y^2 + 25y + 50 = 0$

Carry out the indicated operations, writing results to the nearest hundredth:

7. $5 + 2.646$
8. $-2 - 5.975$
9. $3 - 8.614$
10. $-7 + 4.930$
11. -2.5 ± 1.520
12. $\dfrac{4 \pm 3.162}{2}$

Exercises [A]

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Solve, giving irrational answers in simplest radical form:

1. $2x^2 - 3x - 6 = 0$
2. $2k^2 + 5k = 9$
3. $4y^2 = 8y - 3$
4. $3x^2 = 7x + 6$
5. $7z = 2z^2 + 1$
6. $5x^2 = 20x + 6$
7. $3x^2 - 5x + 1 = 0$
8. $4y^2 = y + 4$

Solve, giving irrational answers to the nearest hundredth:

9. $2x^2 + 7x + 4 = 0$
10. $2z^2 - 2.4z = 2$
11. $4x^2 - 2x - 3 = 0$
12. $4y^2 = 60y - 45$
13. $4c^2 + .8c - 2 = 0$
14. $16y^2 = 8y - 1$
15. $2b^2 - 4b = .4$
16. $3x^2 + 9x = 6$

Solution by Formula

We have noted that all quadratic equations can be arranged in the form

$$ax^2 + bx + c = 0,$$

where a, b, c are integers and $a \neq 0$. It follows, therefore, that any conclusion we can reach about this form, $ax^2 + bx + c = 0$, will be a valid conclusion for *any* quadratic equation. We proceed to solve this equation for x. This is done by the method of the preceding section.

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1. 1; $a + 1$
2. 1; $b - 1$
3. $\frac{1}{4}$; $c + \frac{1}{2}$
4. $\frac{1}{4}$; $d - \frac{1}{2}$
5. $\frac{9}{4}$; $m + \frac{3}{2}$
6. $\frac{9}{4}$; $n - \frac{3}{2}$
7. $\frac{1}{9}$; $p + \frac{1}{3}$
8. $\frac{49}{4}$; $x - \frac{7}{2}$
9. $\frac{9}{64}$; $y + \frac{3}{8}$
10. $\frac{1}{16}$; $w - \frac{1}{4}$
11. $+\sqrt{5}$, $-\sqrt{5}$
12. 0, 5
13. -1, 6
14. $-1\frac{1}{2}$, $6\frac{1}{2}$
15. $\frac{5}{2} \pm \sqrt{17}$
16. 0, 5
17. 0, 4
18. ± 2
19. $9 \pm 2\sqrt{3}$
20. $9 \pm 2\sqrt{3}$
21. 1, 5
22. $3 \pm \sqrt{5}$

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1. -1, 5
2. $6 \pm 4\sqrt{2}$
3. $3 \pm \sqrt{26}$
4. $\frac{-3 \pm \sqrt{17}}{2}$
5. $\frac{7 \pm \sqrt{57}}{2}$
6. $\frac{1 \pm \sqrt{13}}{2}$
7. 2.35
8. 3.98
9. 11.61
10. -11.93
11. 11.70, 0.30
12. 2.73, -9.53
13. -1.06, -3.94
14. 5.45, 0.55
15. 0.54, 7.46
16. -0.48, 1.68
17. 0.54, 1.86
18. -2, 3
19. 0.62, -1.62

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1. $-4 \pm 3\sqrt{2}$
2. -12, 2
3. $-5 \pm \sqrt{31}$
4. $\frac{5 \pm \sqrt{17}}{2}$
5. $\frac{11 \pm \sqrt{137}}{2}$
6. $\frac{-25 \pm 5\sqrt{17}}{2}$
7. 7.65
8. -7.98
9. -5.61
10. -2.07
11. -0.98, -4.02
12. 3.58, 0.42
13. -5.36, 3.36
14. 4, 16
15. -7.28, 0.28
16. -1.0, 0.2
17. -8.65, -0.35
18. -5.85, -0.85

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1. $\frac{3 \pm \sqrt{57}}{4}$
2. $\frac{-5 \pm \sqrt{97}}{4}$
3. $\frac{1}{2}$, $1\frac{1}{2}$
4. $-\frac{2}{3}$, 3
5. $\frac{7 \pm \sqrt{41}}{4}$
6. $2 \pm \frac{1}{5}\sqrt{130}$
7. $\frac{5 \pm \sqrt{13}}{6}$
8. $\frac{1 \pm \sqrt{65}}{8}$
9. -2.78, -0.72
10. -0.57, 1.77
11. -0.65, 1.15
12. 0.79, 14.21
13. -0.81, 0.61
14. $\frac{1}{4}$
15. -0.10, 2.10
16. -3.56, 0.56

Page 470 (middle)

1. $\frac{-3 \pm \sqrt{5}}{2}$
2. $\frac{4 \pm \sqrt{10}}{2}$
3. $\frac{11 \pm \sqrt{145}}{6}$
4. $\frac{1}{4}$, 2
5. $\frac{-13 \pm \sqrt{217}}{8}$
6. $\frac{-22 \pm \sqrt{1294}}{18}$
7. $\frac{8 \pm 4\sqrt{10}}{3}$
8. $1 \pm \sqrt{2}$
9. 0.60, -0.10
10. 1.42, -25.42
11. -4.30, -0.70
12. -0.94, 0.44
13. 1.64, -0.24
14. 4.37, -1.37
15. 1.27, -0.47