

3.8 Exercises

Factor each of the following polynomials. Assume all variables used as exponents are positive integers.

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|----------------------------------------------|------------------------------------------------|-----------------------------|
| 1. $100a^2 - 9b^2$ | 2. $10r^2 + 13r - 3$ | 3. $3p^4 - 3p^3 - 90p^2$ |
| 4. $k^4 - 16$ | 5. $3a^2pq + 3abpq - 90b^2pq$ | 6. $49z^2 - 16$ |
| 7. $225p^2 + 256$ | 8. $x^3 - 1000$ | 9. $6b^2 - 17b - 3$ |
| 10. $k^2 - 6k - 16$ | 11. $18m^3n + 3m^2n^2 - 6mn^3$ | 12. $6r^2 + 19tu - 77u^2$ |
| 13. $2p^2 + 11pq + 15q^2$ | 14. $40p - 32r$ | 15. $9m^2 - 45m + 18m^3$ |
| 16. $4k^2 + 28kr + 49r^2$ | 17. $54m^3 - 2000$ | 18. $mn - 2n - 5m - 10$ |
| 19. $2a^2 - 7a - 4$ | 20. $9m^2 - 30mn + 25n^2$ | 21. $kq - 9q + kr - 9r$ |
| 22. $56k^3 - 875$ | 23. $9r^2 + 100$ | 24. $16z^3x^2 - 32z^2x$ |
| 25. $8p^3 - 125$ | 26. $yx + yw + zx + zw$ | 27. $x^4 - 625$ |
| 28. $2m^2 - mn - 15n^2$ | 29. $p^3 + 64$ | 30. $48y^2z^3 - 28y^3z^4$ |
| 31. $64m^2 - 625$ | 32. $14z^2 - 3zk - 2k^2$ | 33. $12z^3 - 6z^2 + 18z$ |
| 34. $225k^2 - 36r^2$ | 35. $256b^2 - 400c^2$ | 36. $z^2 - zp - 20p^2$ |
| 37. $1000z^3 + 512$ | 38. $64m^2 - 25n^2$ | 39. $10r^2 + 23rs - 5s^2$ |
| 40. $12k^2 - 17kq - 5q^2$ | 41. $24p^3q + 52p^2q^2 + 20pq^3$ | 42. $32x^2 + 16x^3 - 24x^5$ |
| 43. $48k^4 - 243$ | 44. $14x^2 - 25xq - 25q^2$ | 45. $50p^2 - 162$ |
| 46. $y^2 + 3y - 10$ | 47. $12m^2rx + 4mnrx + 40n^2rx$ | 48. $18p^2 + 53pr - 35r^2$ |
| 49. $21a^2 - 5ab - 4b^2$ | 50. $(x - 2y)^2 - 4$ | 51. $(3m - n)^2 - 25$ |
| 52. $(5r + 2s)^2 - 6(5r + 2s) + 9$ | 53. $(p + 8q)^2 - 10(p + 8q) + 25$ | |
| 54. $z^4 - 9z^2 + 20$ | 55. $21m^4 - 32m^2 - 5$ | |
| 56. $(x - y)^3 - (27 - y)^3$ | 57. $z^{4x} - w^{2x}$ | |
| 58. $xy^2z^{-1} + x^2yz^{-2}$ ($z \neq 0$) | 59. $gh^3k^{-2} - g^2h^2k^{-3}$ ($k \neq 0$) | |
| 60. $2a^x + 4a^{2x} + 8a^{3x}$ | 61. $3m^{2k} - 7m^k - 6$ | |
| 62. $2p^{2a} + 10p^a - 28$ | 63. $m^2 - 4m + 4 - n^2 + 6n - 9$ | |
| 64. $x^{2p} - y^{2p}$ | 65. $15c^{4y} + 3c^{2y} - 6c^y$ | |
| 66. $k^2 + 2kp + p^2 - a^2 - 2ab - b^2$ | 67. $8 + (z + w)^3$ | |

Review Exercises Simplify. See Section 3.1.

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|---------------------------------------|------------------------------------|----------------------------------------------------|--------------------------------------------------|
| 68. $\frac{2^4 \cdot 2^{-3}}{2^{-2}}$ | 69. $\frac{3^6 \cdot 3^{-1}}{3^3}$ | 70. $\frac{p^5 p^{-2}}{p^{-1} p^4}$ ($p \neq 0$) | 71. $\frac{2k^{-1}k}{k^2 k^{-2}}$ ($k \neq 0$) |
|---------------------------------------|------------------------------------|----------------------------------------------------|--------------------------------------------------|

Solve each equation. See Section 2.1.

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|-------------------|------------------|-------------------|
| 72. $3x + 1 = 13$ | 73. $5r - 2 = 8$ | 74. $-2q + 3 = 7$ |
| 75. $-a + 4 = 5$ | 76. $4p = 0$ | 77. $-9r = 0$ |
| 78. $-2z + 8 = 9$ | 79. $1 - 3y = 5$ | |

Let x represent the distance between the longer sides as shown in Figure 3.1. Then $x + 8$ is the length of each longer side. The area of a parallelogram is given by $A = bh$ where b is the length of the longer side and h is the distance between the longer sides. Here $b = x + 8$ and $h = x$.

$$\begin{aligned} A &= bh \\ 48 &= (x + 8)x && \text{Let } A = 48, b = x + 8, h = x \\ 48 &= x^2 + 8x && \text{Distributive property} \\ 0 &= x^2 + 8x - 48 && \text{Subtract 48} \\ 0 &= (x + 12)(x - 4) && \text{Factor} \end{aligned}$$

Set each factor equal to 0.

$$\begin{aligned} x + 12 &= 0 && \text{or} && x - 4 &= 0 \\ x &= -12 && \text{or} && x &= 4 \end{aligned}$$

A parallelogram cannot have a negative width, so reject -12 as a solution. The only possible solution is 4, so the distance between the longer sides is 4 meters. The length of the longer sides is $4 + 8 = 12$ meters. ✚

3.9 Exercises

Find all solutions of the following equations by using the zero-factor property. See Example 1.

1. $(x - 2)(x - 3) = 0$
2. $(m + 4)(m + 5) = 0$
3. $(3m - 8)(5m + 6) = 0$
4. $(7y + 4)(3y - 5) = 0$
5. $r(r - 4)(2r + 5) = 0$
6. $3x(3x - 5)(2x + 7) = 0$

Find all solutions of the following equations by factoring. See Examples 2–4.

7. $x^2 - x - 12 = 0$
8. $m^2 + 4m - 5 = 0$
9. $y^2 + 9y + 14 = 0$
10. $15r^2 + 7r = 2$
11. $12x^2 + 4x = 1$
12. $4a^2 + 9a + 2 = 0$
13. $3x^2 - 11x - 20 = 0$
14. $2y^2 - 12 - 4y = y^2 - 3y$
15. $3p^2 + 9p + 30 = 2p^2 - 2p$
16. $4p^2 + 16p = 0$
17. $2a^2 - 8a = 0$
18. $6m^2 - 36m = 0$
19. $-3m^2 + 27m = 0$
20. $-3m^2 + 27 = 0$
21. $-2a^2 + 8 = 0$
22. $4p^2 - 16 = 0$
23. $9x^2 - 81 = 0$
24. $x(x + 3) = 4$
25. $(x + 4)(x - 6) = -16$
26. $(x + 2)(2x + 3) = 10$
27. $(w - 1)(3w + 2) = 4w$
28. $y(y + 1) - 4 = -2(y + 3)$
29. $2x(x + 2) = x(x - 3) - 12$
30. $(r - 5)(r - 3) = 3r(r - 3)$

Solve each of the following word problems by writing a quadratic equation and then solving it. See Example 5.

31. Two numbers have a sum of 6 and a product of -16 . Find the numbers.
32. The sum of two numbers is 10. The sum of their squares is 68. Find the numbers.
33. Two numbers have a sum of 12. The sum of the squares of the numbers is 90. Find the numbers.
34. If two integers are added, the result is 15. If the squares of the integers are added, the result is 125. Find the integers.

69. $(3a^{2z} - 5)^2$ 71. $(x^n - 2)(x^{2n} + 2x^n + 4)$ 73. $(3z^{4y} + 5x^{2y})(9z^{8y} - 15z^{4y}x^{2y} + 25x^{4y})$
 75. $8y(2y - 3 + 4y^2)$ 77. $(x + 2)(y + 4)$ 79. $(y + 2)(y - 1)$ 81. $(3r - z)(2r + 7z)$

Section 3.8 (page 141)

1. $(10a + 3b)(10a - 3b)$ 3. $3p^2(p - 6)(p + 5)$ 5. $3pq(a + 6b)(a - 5b)$ 7. Cannot be factored further
 9. $(6b + 1)(b - 3)$ 11. $3mn(3m + 2n)(2m - n)$ 13. $(2p - 5q)(p + 3q)$
 15. $9m(m - 5 + 2m^2)$ 17. $2(3m - 10)(9m^2 + 30m + 100)$ 19. $(2a - 1)(a - 4)$
 21. $(k - 9)(q + r)$ 23. Cannot be factored further 25. $(2p - 5)(4p^2 - 10p + 25)$
 27. $(x - 5)(x + 5)(x^2 + 25)$ 29. $(p + 4)(p^2 - 4p + 16)$ 31. $(8m + 25)(8m - 25)$
 33. $6z(2z^2 - z + 3)$ 35. $16(4b + 5c)(4b - 5c)$ 37. $8(5z + 4)(25z^2 - 20z + 16)$
 39. $(5r - s)(2r + 5s)$ 41. $4pq(2p + q)(3p + 5q)$ 43. $3(4k^2 + 9)(2k - 3)(2k - 3)$
 45. $2(5p + 9)(5p - 9)$ 47. $4rx(3m^2 + mn + 10n^2)$ 49. $(7a - 4b)(3a + b)$
 51. $(3m - n + 5)(3m - n - 5)$ 53. $(p + 8q - 5)^2$ 55. $(7m^2 + 1)(3m^2 - 5)$
 57. $(z^{2x} + w^x)(z^{2x} - w^x)$ 59. $gh^2k^{-3}(hk - g)$ 61. $(3m^k + 2)(m^k - 3)$
 63. $(m + n - 5)(m - n + 1)$ 65. $3c^x(5c^{3x} + c^x - 2)$
 67. $(2 + z + w)(4 - 2z - 2w + z^2 + 2zw + w^2)$ 69. 9 71. 2 73. {2} 75. {-1}
 77. {0} 79. {-4/3}

Section 3.9 (page 145)

1. {2, 3} 3. {-6/5, 8/3} 5. {-5/2, 0, 4} 7. {-3, 4} 9. {-7, -2}
 11. {-1/2, 1/6} 13. {-4/3, 5} 15. {-6, -5} 17. {0, 4} 19. {0, 9} 21. {-2, 3, 9}
 23. {-3, 3} 25. {-2, 4} 27. {-1/3, 2} 29. {-4, -3} 31. -2 and 8 33. 3, 9
 35. -12, -11 and 11, 12 37. -11, -9 and 9, 11 39. -12 and 6 41. Width, 10 meter length, 14 meters
 43. {-1/2, 6} 45. {-2/3, 4/15} 47. {2, 4} 49. {-3/2, 1/2}
 51. $3m^3$ 53. $-4/(3ab^3)$ 55. 18/24 57. 55/75

Chapter 3 Review Exercises (page 147)

1. Exponent, 3; base, 9 3. Exponent, 2; base 8 5. 243 7. -64 9. 64/27 11. 13^8
 13. m^5 15. m^{5z} 17. 3.45×10^3 19. 1.3×10^{-1} 21. .0038 23. $1/3^8$
 25. -1 27. $25/m^{18}$ 29. $25/z^4$ 31. $2025/(8r^4)$ 33. k^{3r-3} 35. z^{-1-6a} 37. 2
 10⁻⁴ or .0002 39. 4×10^{-5} or .00004 41. 14 43. 29 45. (a) $11k^3 - 3k^2 + 9k$
 (b) Trinomial (c) 3 47. (a) $-5y^4 + 3y^3 + 7y^2 - 2y$ (b) None of these (c) 4 49. (a) 31
 51. (a) -191 (b) 1 53. $13p$ 55. $m^5 + 3m^4$ 57. $x^2 + 11x + 6$ 59. $-4y^3 - 2y^2$
 16y - 14 61. $12m^5 + 11m^3 + 8m^2 - 9m$ 63. $6a^3 - 15a^2 + 7a$ 65. $12y^3$
 67. $-8a^2 - 22a$ 69. $2p^3 + 13p^2 + 17p - 12$ 71. $6z^4 - 8z^3 + 10z^2 + 9z - 2$ 73. n
 4m - 21 75. $6r^2 + 5r - 56$ 77. $9r^4 - 25$ 79. $y^2 + 6yz + 9z^2$ 81. $18y^{2q-1} - 45$
 83. $6z^{2r} - 11x^{2r}z^r - 10x^{4r}$ 85. $5(6p + 5)$ 87. Cannot be factored further
 89. $6ab(2a + 3b^2 - 4a^2b)$ 91. $(x + y)(4 + m)$ 93. $(m + 3)(2 - a)$ 95. $(x - 3)(x - 2)$
 97. $(r + 6)(r + 9)$ 99. $(4p - 1)(p + 1)$ 101. $(4p - 5)(3p + 4)$ 103. $(4a - b)(5a - 2b)$
 105. $(r^2 - 2)(r^2 + 3)$ 107. $(p + 2)^2(p + 3)(p - 2)$ 109. $(3p + 7)(3p - 7)$
 111. $9(4r + 3s + 7)(4r - 3s + 1)$ 113. $(3k - 2)^2$ 115. $(5x - 1)(25x^2 + 5x + 1)$
 117. $(11a^{5m} + 12b^{3m})(11a^{5m} - 12b^{3m})$ 119. $(5r^k + 1)(25r^{2k} - 5r^k + 1)$ 121. {-1, 2/5}
 123. {-4, 2} 125. {-3/2, 1/3} 127. {-3, 3} 129. {1/2, 1} 131. 3 feet
 133. -20/3 or 5 135. -9 or 3 137. $8x^2 - 10x - 3$ 139. 1/125
 141. -14 + 16w - 8w² 143. $-1/(5z^9)$ 145. $1250z^7x^6/9$ 147. $a(6 - m)(5 + m)$
 149. $(2 - a)(4 + 2a + a^2)$ 151. $5y^2(3y + 4)$