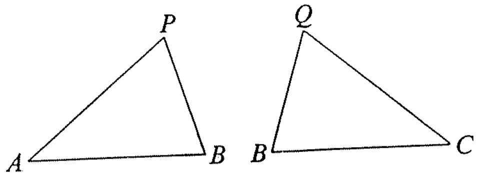
Corresponding Parts

The reason for selecting PA, QC as corresponding sides of the triangles PBA, QBC in the preceding example should be understood. The angles PBA, QBC were proved equal. These angles are, therefore, recognized as corresponding parts of the triangles. It follows, then, that the sides PA, QC, which are opposite these angles, are also corresponding parts.



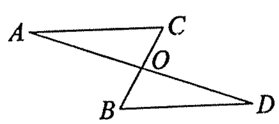
The other pairs of corresponding parts listed in step 9 are selected in the same way. The fact that $\angle A = \angle C$ tells us that PB and QB are corresponding parts of the two triangles. Why are $\angle P$, $\angle Q$ corresponding parts?

In naming congruent triangles, the vertices of one triangle should be given in the same order as the corresponding vertices of the other triangle. If we say that $\triangle PMX$ is congruent to $\triangle AYT$, then P, M, X should correspond to A, Y, T respectively. When this procedure is followed, the order of the letters is a guide in the selection of corresponding parts of the triangles.

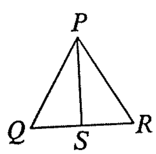
Exercises [A]

In each of the following exercises, prove that the triangles in the diagrams are congruent, and name the remaining pairs of equal corresponding parts.

1. AOD and BOC are straight lines, $AO = OD$, and $BO = OC$.

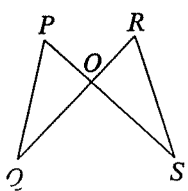


Ex. 1

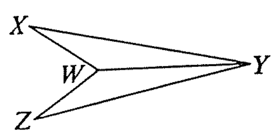


Ex. 2

2. S is the midpoint of QR, and PS is perpendicular to QR.
3. POS and QOR are straight lines, $PO = OR$, and $\angle P = \angle R$.



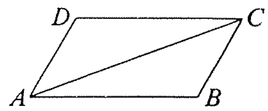
Ex. 3



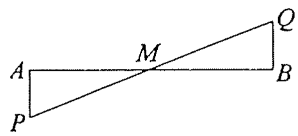
Ex. 4

4. $YX = YZ$ and YW bisects $\angle XYZ$.

5. $\angle CAB = \angle ACD$ and $\angle CAD = \angle BCA$.

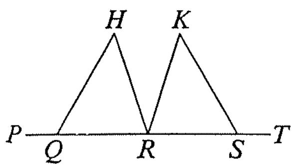


Ex. 5

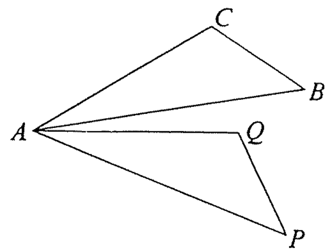


Ex. 6

6. PMQ is a straight line which bisects AB at M. PA and QB are both perpendicular to AB.
7. $PQRST$ is a straight line, $QR = RS$, $\angle HRQ = \angle KRS$, and $\angle HQP = \angle KST$.

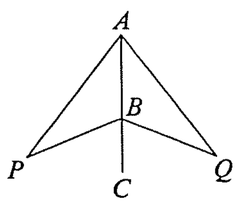


Ex. 7

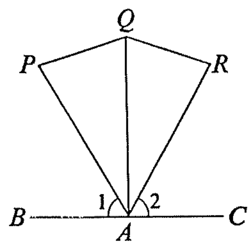


Ex. 8

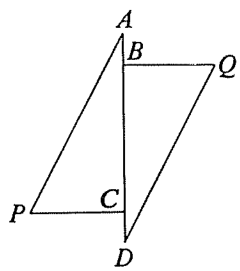
8. $AB = AP$, $AC = AQ$, and $\angle BAP = \angle CAQ$.
9. ABC is a straight line bisecting $\angle PAQ$, and $\angle PBC = \angle QBC$.



Ex. 9



Ex. 10



Ex. 11

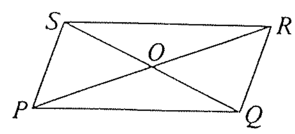
10. AQ is perpendicular to BAC, $\angle 1 = \angle 2$, and $AP = AR$.
11. $ABCD$ is a straight line, in which $AB = CD$. PC and QB are both perpendicular to $ABCD$, and $\angle A = \angle D$.

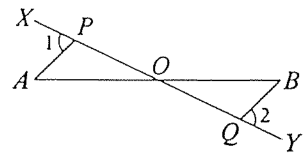
Most problems in geometry require us to prove something more specific than was the case in the preceding exercises. Generally we are asked to prove that a pair of line segments or a pair of angles are equal. To do this it is usually necessary to prove the congruence of a pair of triangles in which these specified line segments or angles are equal corresponding parts.

Exercises [A]

1. O is the midpoint of SQ and of PR. Prove that $SP = RQ$.

Ex. 1



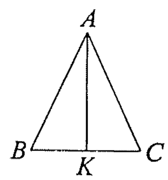


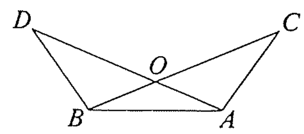
Ex. 2

2. $XPOQY$ and AOB are straight lines. $PO = OQ$ and $\angle 1 = \angle 2$. Prove that $\angle A = \angle B$.

3. $AB = AC$, AK is the bisector of $\angle BAC$, meeting BC at K. Prove that $\angle B = \angle C$.

Ex. 3



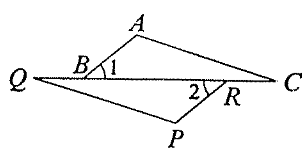


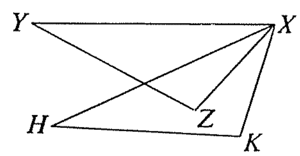
Ex. 4

4. $BC = AD$ and O is a point on each line such that $DO = CO$. Prove that $DB = CA$.

5. $QBRC$ is a straight line with $QB = RC$, $\angle Q = \angle C$, and $\angle 1 = \angle 2$. Prove that $AB = RP$.

Ex. 5



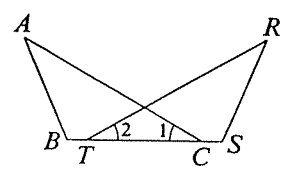


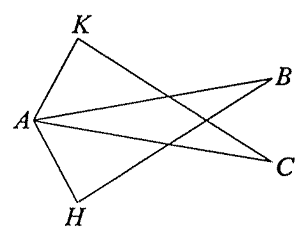
Ex. 6

6. $YX = HX$, $\angle Y = \angle H$, and $\angle HXY = \angle KXZ$. Prove that $\angle Z = \angle K$.

7. $BTCS$ is a straight line, $BT = CS$; $AC = RT$, and $\angle 1 = \angle 2$. Prove that $AB = RS$.

Ex. 7



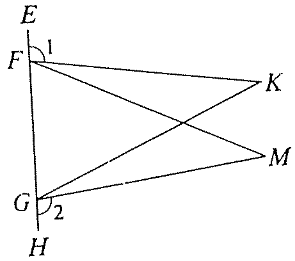


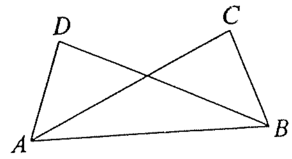
Ex. 8

8. $AB = AC$, $AH = AK$, $\angle BAK = \angle CAH$. Prove that $\angle K = \angle H$.

9. $EFGH$ is a straight line, $FK = GM$, and $\angle 1 = \angle 2$. Prove that $\angle K = \angle M$.

Ex. 9



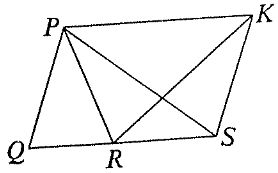


Ex. 10

10. $\angle CAB = \angle DBA$ and $\angle CAD = \angle DBC$. Prove that $AC = BD$.

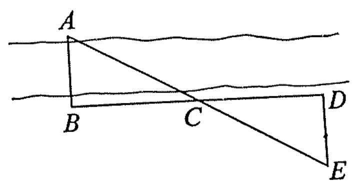
11. QRS is a straight line, $PQ = PR$, $\angle QPR = \angle SPK$, and $\angle Q = \angle PRK$. Prove that $PS = PK$.

Ex. 11



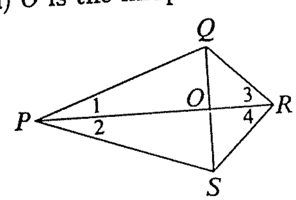
12. Draw a triangle ABC in which $\angle B = \angle C$. Let the bisector of $\angle B$ meet AC at H and let the bisector of $\angle C$ meet AB at K. Prove that $\angle AHB = \angle AKC$.

13. Justify the following field construction for finding the width of a river without crossing it.



Stand opposite a point A on the far side of the river at which there is some kind of landmark, say a tree. Your position as shown in the diagram is at B. From B walk a convenient distance along the bank at right angles to AB, to a point C. Mark the point C and continue to a point D, making $CD = BC$. Then turn and walk at right angles to CD until you reach a point E which is in line with A and C. Then DE is equal to the width of the river.

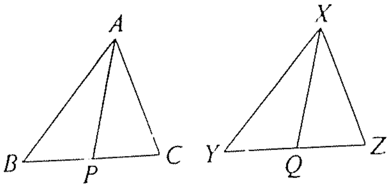
Example [B] In the diagram, POR and QOS are straight lines, $\angle 1 = \angle 2$, and $\angle 3 = \angle 4$. Prove (a) O is the midpoint of QS, (b) $PR \perp QS$.



Exercises [B]

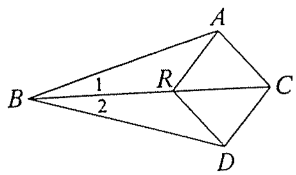
1. In the triangles ABC and XYZ, $\angle BAC = \angle YXZ$, $AB = XY$, and $AC = XZ$. If P and Q are the midpoints of BC and YZ respectively, prove that $AP = XQ$.

2. Two triangles ABC and DEF are such that $\angle B = \angle E$, $\angle C = \angle F$, $BC = EF$, and P and Q are points on AC and DF respectively such that $AP = DQ$. Prove that $BP = EQ$.

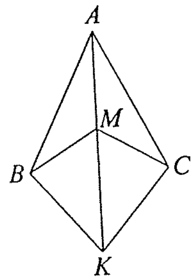


Ex. 1

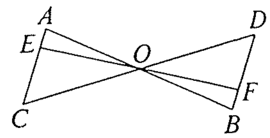
3. In the first diagram below, BRC is a straight line, $AB = BD$, and $\angle 1 = \angle 2$. Prove that $\angle RAC = \angle RDC$.



Ex. 3



Ex. 4



Ex. 5

4. AMK is a straight line bisecting $\angle BAC$. $\angle AMB = \angle AMC$. Prove that $BK = CK$.

5. AOB, COD, EOF, AEC, and BFD are straight lines. $AO = OB$ and $CO = OD$. Prove that $AE = BF$.

6. A triangle ABC is such that $AB = AC$. AB is extended to P and AC is extended to Q so that $BP = CQ$. Join PC and QB and prove that $\triangle ACP \cong \triangle ABQ$. Note the corresponding parts of these triangles that are equal, and hence prove that $\triangle BCP \cong \triangle CBQ$.

7. A triangle ABC is such that $\angle B = \angle C$. The bisector of $\angle B$ meets AC at H and the bisector of $\angle C$ meets AB at K. Prove that $AH = AK$.

8. M is the midpoint of a line segment AB. Through M is drawn a line segment XY such that XA and YB are both perpendicular to AB. Prove that $AY = BX$.

9. $\triangle ABC$ has $AB = AC$. X is a point on AC, and Y is a point on AB, such that $AX = AY$.

   Prove: (a) $BX = CY$, (b) $\angle ABC = \angle ACB$.