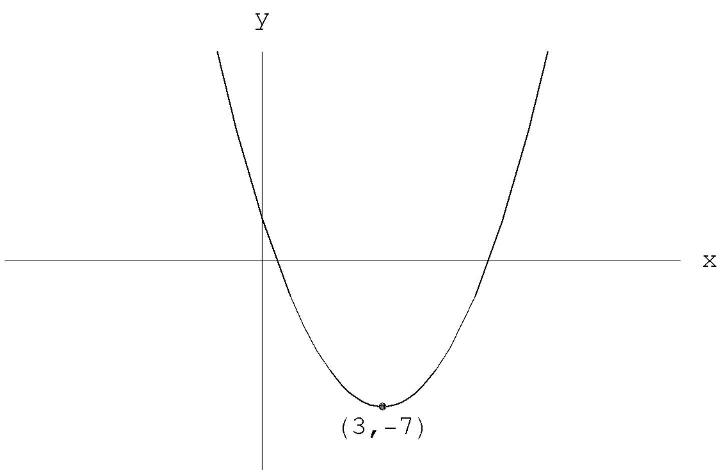
Proving that $f(x_i)$ is a maximum/minimum at x_i.

Consider the function *f* defined as $f(x) = x^2 - 6x + 2$. Completing the square yields

$$f(x) = (x - 3)^2 - 7 \quad (1)$$

Inspecting EQ1, we see that the minimum value of the function is -7 and that it occurs when $x = 3$. That is, the point $(3, -7)$ is the lowest point on the function's graph.



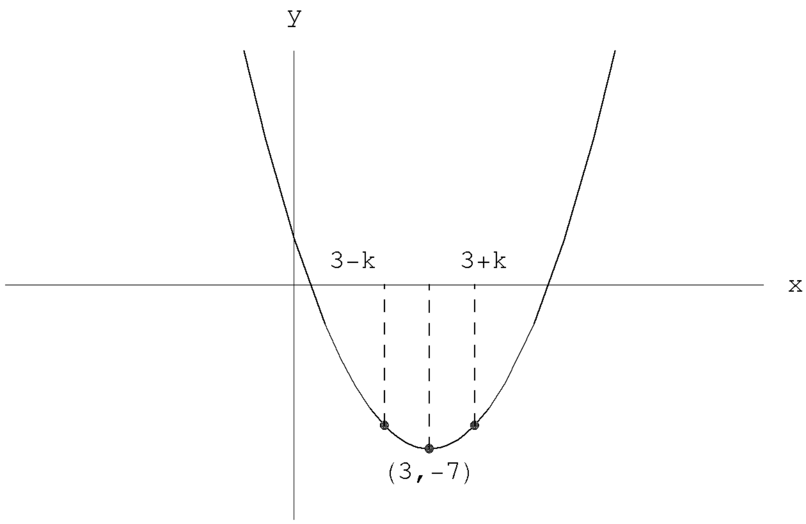
Suppose we wish to prove that -7 is the minimum value of the function. The stategy is to argue that for any *x* other than $x = 3$, the value of the function is greater than -7. We will show that

for any $x > 3$, $f(x) > -7$

and that

for any $x < 3$, $f(x) > -7$.

Here's the geometry



Here's the algebra.

Case 1. Suppose $x > 3$. Then $x = 3 + k, \; k \in \mathbb{R}, k > 0$. Now

$$\begin{aligned} f(3+k) &= (3+k-3)^2 - 7 \\ &= k^2 - 7 \\ &> -7 \end{aligned}$$

since k^2 is a positive number.

Case 2. Suppose $x < 3$. Then $x = 3 - k, \; k \in \mathbb{R}, k > 0$. Now

$$\begin{aligned} f(3+k) &= (3-k-3)^2 - 7 \\ &= k^2 - 7 \\ &> -7 \end{aligned}$$

since k^2 is a positive number.

Therefore, if $x \neq 3$, $f(x) > -7$. Since $f(3) = -7$, we see that -7 is the minimum value attained by the function.