Terminology and concepts

■ Function

There are several good definitions of function.

A **function** is a rule or correspondence that assigns exactly one member of a set $\mathcal{B}$ to each member of a set $\mathcal{A}$. The set $\mathcal{A}$ is called the domain of the function. The set $\mathcal{B}$ is called the range of the function.

More formally,

A **function** is a set of ordered pairs such that each first component is paired with one and only one second component. That is, **F** is a function if $(x_1, y_1) \in F$ and $(x_1, y_2) \in F$ implies that $y_1 = y_2$.

A **function** is a set of ordered pairs such that each first component is paired with one and only one second component. That is, **F** is a function if $(x_1, x_2) \in F$ and $(x_2, y_2) \in F$ and $x_1 = x_2$ implies that $y_1 = y_2$.

A **function** is a set of ordered pairs in which no first element occursmore than once.

■ Domain, codomain, and range

The **domain** of a function is the set of objects for which the function is defined.

The **codomain** set within which the values of a function lie.

The **range** of a function is the set of values that the function actually takes.

■ Notation and language

The function is represented by a single letter, for example *f*. The value of the function when evaluated at *x* is $f(x)$. Sometimes, $f(x)$ is called *the image of x under f.* *x* is called the argument. *x* is the independent variable, $f(x)$ is the dependent variable. *x* is the input, $f(x)$ is the output. Often, a function is defined by a statement such as $f(x) = 2x^2 + 3$; this defines the function *f* by giving the rule by which $f(x)$ is computed. Alternatively, we can define a function by a statement such as "$f : \mathcal{A} \mapsto \mathcal{B}$ such that $f(x) = 2x^2 + 3$"; this says that *f* is a function from a set $\mathcal{A}$ into a set $\mathcal{B}$, and the rule that associates $f(x) \in \mathcal{B}$ with $x \in \mathcal{A}$ is $2x^2 + 3$. Sometimes a function is called a **map**, thus a function is said to map elements of one set into elements of another set.

Understanding a function

To understand a function, we look at several of its characteristics which may include those mentioned below. Nearly always, a sketch or a mental picture showing these characteristics is valuable.

■ 1. Domain

■ 2. Range

■ 3. Zeros

■ 4. Asymptotes

■ 5. Extreme values (maximum, minimum)

■ 6. Monotonicity (increasing, decreasing)

■ 7. Symmetry

■ 8. Rate of change

■ 9. One-to-One (injection)

■ 10. Onto (surjection)

■ 11. One-to-one onto (bijection)

Algebra of functions

■ Composition