

Some important sets

$\mathbb{N} = \{1, 2, 3, \dots\}$	The set of natural numbers.
$\mathbb{W} = \{0, 1, 2, 3, \dots\}$	The set of whole numbers.
$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	The set of integers.
$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$	The set of positive integers.
$\mathbb{Z}^- = \{-1, -2, -3, \dots\}$	The set of negative integers.
$\mathbb{Q} = \left\{\frac{p}{q}, p \in \mathbb{Z}, q \neq 0 \in \mathbb{Z}\right\}$	The set of rational numbers.
\mathbb{R}	The set of real numbers
\mathbb{C}	The set of complex numbers

Indicating the membership of a set

■ Set builder notation

- [EX1] The set of all real numbers greater than 5 is written

$$\{x \in \mathbb{R} : x > 5\}$$

The brackets $\{ \}$ indicate a set. The colon $:$ is read "such that" and it is followed by a condition that an object must satisfy in order to be included in the set. Thus,

$\{x \in \mathbb{R} : x > 5\}$ is read "the set of all numbers x such that x is greater than 5".

Sometimes, we write $\{x : x > 5\}$. In such cases we understand that the x preceding the $:$ to be any real number.

- [EX2] The set of all rational numbers less than or equal to 5.
 $\{x \in \mathbb{Q} : x \leq 5\}$
- [EX3] The set of real numbers less than or equal to 5 and greater than -6 .
 $\{x : x \leq 5 \wedge x > -6\}$ or equivalently $\{x : -6 < x \leq 5\}$
- [EX4] The set of real numbers less than or equal to 5 or greater than -6 .
 $\{x : x \leq 5 \vee x > -6\}$

- [EX5] The set of all ordered pairs that lie on the line $y = 3x - 2$.

$$\{(x, y) : x \in \mathbb{R} \wedge y = 3x - 2\}$$

- [EX6] The set of all non-negative integers

$$\{x \in \mathbb{Z} : x \geq 0\}$$

■ **Note:**

Other symbols used for the phrase "such that" include \ni and $|$. The following all indicate the set of real numbers between 7 and 11.

$$\{x \in \mathbb{R} : 7 \leq x \leq 11\}$$

$$\{x \in \mathbb{R} \ni 7 \leq x \leq 11\}$$

$$\{x \in \mathbb{R} | 7 \leq x \leq 11\}$$

Interval notation

Suppose we wish to indicate the portion of the real number line that includes the numbers 2 and 5 and all the real numbers between them. That is, we wish to indicate the **interval** from 2 to 5 including its endpoints. This is how we do that.

[2, 5] The portion of the real number line from 2 to 5 including the endpoints 2 and 5.

[2, 5] Includes all the numbers in $\{x \in \mathbb{R} \ni 2 \leq x \leq 5\}$.

Suppose we wish to indicate the portion of the real number line that includes all the numbers between 2 and 5 but does not include 2 and does not include 5. That is, we wish to indicate the **interval** from 2 to 5 excluding its endpoints. This is how we do that.

(2, 5) The portion of the real number line from 2 to 5 excluding the endpoints 2 and 5.

(2, 5) Includes all the numbers in $\{x \in \mathbb{R} \ni 2 < x < 5\}$

An interval that includes its endpoints, written with two square brackets, is called a **closed interval**. An interval that excludes its endpoints, written with two parenthesis, is called an **open interval**.

An interval may be open on one end, but closed at the other. For example:

(2, 5] The portion of the real number line from 2 to 5 excluding 2 but including 5.

(2, 5] Includes all the numbers in $\{x \in \mathbb{R} \ni 2 < x \leq 5\}$

[2, 5) The portion of the real number line from 2 to 5 including 2 but excluding 5.

[2, 5) Includes all the numbers in $\{x \in \mathbb{R} \ni 2 \leq x < 5\}$

We can also indicate infinite intervals; that is, intervals that are unbounded at one or both ends. For examples:

$(2, \infty +)$ The portion of the real number line to the right of 2. $\{x \in \mathbb{R} \ni x > 2\}$.

$(-\infty, 2)$ The portion of the real number line to the left of 2. $\{x \in \mathbb{R} \ni x < 2\}$

$(-\infty, \infty +)$ The real number line. \mathbb{R} .

■ Further examples

[EX1] Use interval notation to indicate the interval from -12 to 6 , including -12 and 6 , but excluding 2 .

$[-12, 2) \cup (2, 6]$

[EX2] Use interval notation to indicate the interval from -12 to 6 , including -12 and 6 , or the interval from 20 to 31 , including the endpoints..

$[-12, 6] \cup [20, 31]$