Thm: If $a^2 = b$, $b \geq 0$

Then $a = \pm\sqrt{b}$

[EX 1] $x^2 = 7$

$\therefore x = \pm\sqrt{7}$

[EX 2] $(x-3)^2 = 7$

$\Rightarrow x - 3 = \pm\sqrt{7}$

NOTE: this comes from the 1st line by an application of the theorem. The "a" in the theorem is just replaced by the number $x-3$.

$\therefore x = 3 \pm \sqrt{7}$

meaning: $x = 3 + \sqrt{7}$ OR $x = 3 - \sqrt{7}$

SOLVE $x^2 + bx + c = 0$, $b \neq 0$, $c \neq 0$, by using the theorem above. The idea is we want to rewrite $x^2 + bx + c = 0$ so that it matches the form $a^2 = b$.

The technique called "completing the square" enables us to rewrite $x^2 + bx + c = 0$ in the form $a^2 = b$.

$\longrightarrow$

completing the square

We illustrate the method by solving $x^2-5x+6=0$. You already know the solution of this equation is $x=2$ or $x=3$.

$$x^2-5x+6=0$$

Step ① $\quad x^2-5x = -6$

[Get all and only the terms with the unknown on one side, all the other terms on the other side.]

STEP ②

$$x^2-5x+\left[\tfrac{5}{2}\right]^2 = -6+\left[\tfrac{5}{2}\right]^2$$

[(one half the coefficient of x)^2 added to the LHS makes the LHS a perfect square. Of course you must add the same item to the RHS.]

STEP ③

$$\left(x-\tfrac{5}{2}\right)^2 = -6+\left[\tfrac{5}{2}\right]^2$$

[one half the coefficient of x appears in the factored form on the LHS. It will <u>always</u> factor like this. Your choice of $\left[\frac{5}{2}\right]^2$ in Step to guarantees this.]

Step ④ $\quad x-\frac{5}{2} = \pm\sqrt{-6+\left[\frac{5}{2}\right]^2}$

[This is the theorem we mentioned and proved above]

$\longrightarrow$

step ⑤ $x = \frac{5}{2} \pm \sqrt{-6 + \left[\frac{5}{2}\right]^2}$

step ⑥ simplify answer

$$x = \frac{5}{2} \pm \sqrt{-6 + \frac{25}{4}}$$

$$= \frac{5}{2} \pm \sqrt{\frac{-24+25}{4}}$$

$$= \frac{5}{2} \pm \sqrt{\frac{1}{4}}$$

$$= \frac{5}{2} \pm \frac{1}{2}$$

$$\boxed{\therefore \quad x = \frac{5 \pm 1}{2}}$$ DONE.

Let's see if we got the answer we already knew was correct

$\frac{5+1}{2} = \frac{6}{2} = 3$ as expected

$\frac{5-1}{2} = \frac{4}{2} = 2$ as expected

Yahoo!

→

[Ex 3] Solve $x^2 - 7x + 10 = 0$

— NOTE: you already know solution is $x = 2$ or $x = 5$.

Solution

$$x^2 - 7x + 10 = 0$$

$$x^2 - 7x = -10$$

$$x^2 - 7x + \left[\frac{7}{2}\right]^2 = -10 + \left[\frac{7}{2}\right]^2$$

$$\left(x - \frac{7}{2}\right)^2 = -10 + \frac{49}{4}$$

$$\left(x - \frac{7}{2}\right)^2 = \frac{9}{4}$$

we might as well get the arith. out of the way now.

$$x - \frac{7}{2} = \pm\sqrt{\frac{9}{4}}$$

$$x = \frac{7}{2} \pm \frac{3}{2}$$

$$= \frac{7 \pm 3}{2}$$

$$= \frac{10}{2} \text{ OR } \frac{4}{2}$$

$$\boxed{\therefore\ x = 5 \text{ or } 2}$$ answer

Summary

you can solve <u>any</u> quadratic EQN in one unknown by the method of completing the square. It <u>always</u> works (easy to prove this).

The general idea is rewrite the quadratic in the form $a^2 = b$, then use the thm

$$a^2 = b \Rightarrow a = \pm\sqrt{b}, \quad b \geq 0.$$

—————— END OF CLASS ——————

How to handle a quadratic eqn of the form

$$ax^2 + bx + c = 0, \quad a \neq 1, \quad a \neq 0, b \neq 0, c \neq 0 \; ?$$

Note the examples so far have all had $a = 1$, as in

$1x^2 - 5x + 6 = 0$.

— Answer. Divide both sides of EQN by a. Then you will have a coefficient of 1 on x^2. The proceed as illustrated above.

⟶

[Ex 4] Solve $3x^2 - 11x + 6 = 0$

$$3x^2 - 11x + 6 = 0$$

$$x^2 - \frac{11}{3}x + 2 = 0$$ div by 3

$$x^2 - \frac{11}{3}x = -2$$

$$x^2 - \frac{11}{3}x + \frac{121}{36} = -2 + \frac{121}{36}$$

half of $\frac{11}{3} = \frac{11}{2 \cdot 3} = \frac{11}{6}$

$\left(\frac{11}{6}\right)^2 = \frac{121}{36}$

$$\left(x - \frac{11}{6}\right)^2 = \frac{49}{36}$$

$$x - \frac{11}{6} = \pm\sqrt{\frac{49}{36}}$$

$$x - \frac{11}{6} = \pm\frac{7}{6}$$

$$x = \frac{11}{6} \pm \frac{7}{6}$$

$$= \frac{18}{6} \text{ or } \frac{4}{6}$$

$$\boxed{\therefore \quad x = 3 \text{ or } \frac{2}{3}}$$

DONE