[06-02-06-T7]

Rational Expressions

Read this.

Thm. $-\frac{a}{b} = \frac{-a}{b}$, provided $b \neq 0$.

Proof.

$$
\begin{aligned}
-\frac{a}{b} &= (-1)\,\frac{a}{b} \\
&= (-1)\cdot a \cdot \frac{1}{b} \\
&= -a \cdot \frac{1}{b} \\
&= \frac{-a}{b}
\end{aligned}
$$

Discussion: the theorem says that the quotient of a negative number divided by a positive number is negative. That is, $\frac{-a}{b} = -a \div b = -\frac{a}{b}$. For example $\frac{-5}{8} = -\frac{5}{8}$. Or, $\frac{-5}{8} = -5 \div 8 = -0.625 = -\frac{5}{8}$.

You can use this idea to work with an expression such as $\frac{x+y}{3} - \frac{2x+3}{2}$. Here's how:

$$
\begin{aligned}
&\frac{x+y}{3} - \frac{2x+3}{2} \\
&= \frac{2(x+y)}{6} \boldsymbol{-} \boldsymbol{\frac{3(2x+3)}{6}} \\
&= \frac{2(x+y)}{6} \boldsymbol{+} \boldsymbol{\frac{-3(2x+3)}{6}} \\
&= \frac{2x+2y}{6} + \frac{-6x-9}{6} \\
&= \frac{-4x+2y-9}{6}
\end{aligned}
$$

The part in **bold** is where the theorem was used.

If you wish, the result $\frac{-4x+2y-9}{6}$ could be rewritten as $-\frac{4x-2y+9}{6}$, using the theorem again.

Do this.

■ A. Write as a single fraction in lowest terms (fully reduced)

[1] $\frac{-2\,a}{3} - \frac{-a+b}{5}$

[2] $\frac{x+y}{3} - \frac{2\,x+3}{2}$

[3] $-\frac{x}{3} - \frac{2\,x-5}{4}$

[4] $\frac{x-2}{3\,a} - \frac{x+7}{2\,a}$

[5] $\frac{x-2}{5} - \frac{x-2}{2}$

Answers

[1] $\frac{-7a-3b}{15}$

[2] $\frac{-4x+2y-9}{6}$

[3] $\frac{15-10x}{12}$

[4] $-\frac{x+25}{6a}$

[5] $\frac{3x+6}{10}$