2 Simplifying Fractions

Fractions having several terms in the numerator or denominator can often be simplified too.

Factor the numerator. →

$$\frac{4x-8}{8} = \frac{\overset{1}{\cancel{4}}(x-2)}{\underset{2}{\cancel{8}}} = \frac{x-2}{2}$$

WARNING! In examples like these, write out the factors first.

Correct →

$$\frac{4x-8}{8} = \frac{\overset{1}{\cancel{4}}(x-2)}{\underset{2}{\cancel{8}}} = \frac{x-2}{2}$$

Not Correct →

$$\frac{\overset{1}{\cancel{4x-8}}}{\underset{1}{\cancel{8}}} = \frac{4x-1}{1} = 4x-1$$

Here are a few more examples to study.

EXAMPLE 1

$$\frac{x^2-3x}{2x-6} = \frac{x(x-3)}{2(x-3)}$$

$$= \frac{\overset{1}{\cancel{x(x-3)}}}{\underset{1}{\cancel{2(x-3)}}}$$ ← Divide the numerator and denominator by $(x-3)$.

$$= \frac{x}{2}$$

EXAMPLE 2

$$\frac{3t-15}{t^2-25} = \frac{3(t-5)}{(t-5)(t+5)} = \frac{3}{t+5}$$

EXAMPLE 3

$$\frac{a^2-2a+1}{a^2-a} = \frac{(a-1)(a-1)}{a(a-1)}$$

$$= \frac{a-1}{a}$$

Classroom Practice

Simplify.

1. $\frac{3a + 3b}{3a}$
2. $\frac{x + 1}{2x + 2}$
3. $\frac{y - 4}{2y - 8}$
4. $\frac{x^2 - 1}{x - 1}$
5. $\frac{x^2 + x}{x + 1}$
6. $\frac{a - b}{2a - 2b}$
7. $\frac{a^2 - 4}{a + 2}$
8. $\frac{x^2 + 2x + 1}{x^2 - 1}$

9. For what values of x does the fraction $\frac{5}{x^2 - 4}$ have no meaning?

Written Exercises

Simplify.

A

1. $\frac{x + 1}{3x + 3}$
2. $\frac{x - 1}{2x - 2}$
3. $\frac{3p + 3q}{7p + 7q}$
4. $\frac{2a - 2b}{2a + 2b}$
5. $\frac{x^2 + 2x}{x + 2}$
6. $\frac{a + 1}{a^2 + a}$
7. $\frac{3y}{9y^2 - 6y}$
8. $\frac{6a + 6b}{9a + 9b}$
9. $\frac{2c + 6}{c^2 - 9}$
10. $\frac{a^2 - 4}{a - 2}$
11. $\frac{2x + 10}{x^2 - 25}$
12. $\frac{x^2 - y^2}{x + y}$
13. $\frac{n^2 - 9}{n^2 + 3n}$
14. $\frac{y^2 - 4y}{2y - 8}$
15. $\frac{2xy - 6x}{y^2 - 3y}$
16. $\frac{2x - 4xy}{1 - 2y}$
17. $\frac{5k + 10}{(k + 2)^2}$
18. $\frac{(m - 4)^2}{3m - 12}$
19. $\frac{6u + 6v}{u^2 - v^2}$
20. $\frac{g^2 - h^2}{5g - 5h}$
21. $\frac{12x^2y}{3xy^2 + 6x^2y}$
22. $\frac{3x - 9xy}{1 - 3y}$
23. $\frac{x^2 + 6x + 9}{x^2 - 9}$
24. $\frac{y^2 - 4y + 4}{y^2 - 4}$

B

25. $\frac{2x^2 - 50}{2x + 10}$
26. $\frac{2ab^2 + 2a^2b}{4a^2 - 4b^2}$
27. $\frac{3x^2y}{6x^2y + 3xy^2}$
28. $\frac{x^2 - 8x + 15}{x^2 - 2x - 15}$
29. $\frac{x^2 - x - 6}{x^2 + 5x + 6}$
30. $\frac{x^2 - 6x + 8}{x^2 - x - 2}$
31. $\frac{y^2 - 4y - 5}{y^2 - 2y - 3}$
32. $\frac{t^2 + 7t + 10}{t^2 - t - 6}$
33. $\frac{z^2 - 12z + 20}{z^2 + 4z - 12}$

3 The −1 Factor

You know that 1 is a factor of every number, even though the factor 1 is not written very often.

$2 = 1 \cdot 2$ $\qquad x = 1 \cdot x$ $\qquad a - 1 = 1(a - 1)$

You can also write -1 as a factor of every number.

$2 = -1(-2)$ $\qquad x = -1(-x)$ $\qquad a - 1 = -1(-a + 1)$ *or* $a - 1 = -1(1 - a)$

Now see how useful this -1 factor can be in simplifying fractions.

EXAMPLE 1 $\dfrac{a-1}{1-a} = \dfrac{a-1}{-1(a-1)} = \dfrac{\overset{1}{\cancel{a-1}}}{-1\underset{1}{\cancel{(a-1)}}} = \dfrac{1}{-1} = -1$

EXAMPLE 2 $\dfrac{b-a}{2a-2b} = \dfrac{b-a}{2(a-b)} = \dfrac{-1(a-b)}{2(a-b)} = \dfrac{-1}{2} = -\dfrac{1}{2}$

EXAMPLE 3 $\dfrac{x^2-9}{3-x} = \dfrac{(x-3)(x+3)}{3-x} = \dfrac{(x-3)(x+3)}{-1(x-3)} = \dfrac{x+3}{-1} = -x-3$

Classroom Practice

Complete.

1. $3 = -1(\underline{\ ?\ })$
2. $7 = -1(\underline{\ ?\ })$
3. $17 = -1(\underline{\ ?\ })$
4. $2x = -1(\underline{\ ?\ })$
5. $-x - y = -1(\underline{\ ?\ })$
6. $-x + y = -1(\underline{\ ?\ })$
7. $a + b = (\underline{\ ?\ })(-a - b)$
8. $1 - x = (\underline{\ ?\ })(x - 1)$
9. $2 - y = -1(\underline{\ ?\ })$
10. $a - b = -1(\underline{\ ?\ })$
11. $3 + x = -1(\underline{\ ?\ })$
12. $x - 2 = -1(\underline{\ ?\ })$
13. $2a - b = -1(\underline{\ ?\ })$
14. $-x - 4y = -1(\underline{\ ?\ })$
15. $x^2 - 4 = -1(\underline{\ ?\ })$
16. $9 - a^2 = -1(\underline{\ ?\ })$
17. $-b^2 + 1 = -1(\underline{\ ?\ })$
18. $m^2 + 2 = -1(\underline{\ ?\ })$

Written Exercises

Write the expression with a factor of −1.

A

1. $-2x$
2. $-3y$
3. $-a - b$
4. $-7 - 4x$
5. $2 - 3y$
6. $6 - 5b$
7. $3 - 8m$
8. $16 - r$
9. $25 - v$
10. $3 - x^2$
11. $-4 - y^2$
12. $a^2 - b^2$

Simplify.

13. $\frac{a-b}{b-a}$
14. $\frac{c-2}{2-c}$
15. $\frac{x-y}{y-x}$
16. $\frac{a-c}{c-a}$
17. $\frac{1-a}{a-1}$
18. $\frac{b-1}{1-b}$
19. $\frac{m-n}{n-m}$
20. $\frac{a-6}{6-a}$
21. $\frac{2x+2y}{-y-x}$
22. $\frac{3b+6a}{-b-2a}$
23. $\frac{5x-5y}{5y-5x}$
24. $\frac{1-b}{2b-2}$
25. $\frac{a^2-1}{1-a}$
26. $\frac{m^2-1}{1-m}$
27. $\frac{1-r}{r^2-1}$
28. $\frac{a^2-9}{3-a}$
29. $\frac{2-c}{c^2-4}$
30. $\frac{3y^2-3}{3-3y}$
31. $\frac{2x^2-50}{10-2x}$
32. $\frac{-a-b}{b^2-a^2}$

B

33. $\frac{8-4t}{t^2+t-6}$
34. $\frac{5-5x}{x^2+3x-4}$
35. $\frac{p^2-4p-21}{14-2p}$
36. $\frac{z^2-6z+9}{6-2z}$
37. $\frac{r^2-7r+12}{9-3r}$
38. $\frac{8-4a}{a^2-5a+6}$

SELF-TEST

Simplify.

1. $\frac{24}{21x}$
2. $\frac{-6a}{18a}$
3. $\frac{-12r^2s}{-9r^2s^2}$ *(9-1)*
4. $\frac{5a-5b}{a-b}$
5. $\frac{9-m^2}{3+m}$
6. $\frac{6x+6y}{9x+9y}$ *(9-2)*
7. $\frac{x-3y}{3y-x}$
8. $\frac{4m-4n}{2n-2m}$
9. $\frac{2r-8s}{-r+4s}$ *(9-3)*

Written Exercises, page 301

1. $\frac{1}{3}$ **3.** $\frac{3}{7}$ **5.** x **7.** $\frac{1}{3y-2}$ **9.** $\frac{2}{c-3}$
11. $\frac{2}{x-5}$ **13.** $\frac{n-3}{n}$ **15.** $\frac{2x}{y}$
17. $\frac{5}{k+2}$ **19.** $\frac{6}{u-v}$ **21.** $\frac{4x}{y+2x}$
23. $\frac{x+3}{x-3}$ **25.** $x-5$ **27.** $\frac{x}{2x+y}$
29. $\frac{x-3}{x+3}$ **31.** $\frac{y-5}{y-3}$ **33.** $\frac{z-10}{z+6}$

Written Exercises, page 303

1. $-1(2x)$ **3.** $-1(a+b)$
5. $-1(3y-2)$ **7.** $-1(8m-3)$
9. $-1(v-25)$ **11.** $-1(4+y^2)$ **13.** -1
15. -1 **17.** -1 **19.** -1 **21.** -2
23. -1 **25.** $-a-1$ **27.** $-\frac{1}{r+1}$
29. $-\frac{1}{c+2}$ **31.** $-x-5$ **33.** $-\frac{4}{t+3}$
35. $-\frac{p+3}{2}$ **37.** $-\frac{r-4}{3}$

Written Exercises, page 305 **1.** $\frac{5}{8}$ **3.** $\frac{2}{3}$
5. $\frac{3}{5}$ **7.** $\frac{2}{1}$ **9.** $\frac{3}{2}$ **11.** $\frac{3}{2}$ **13.** $\frac{1}{7}$ **15.** $\frac{1}{5}$
17. $\frac{3}{1}$

Written Exercises, page 307 **1.** 20 boys **3.** 48 tails **5.** 25 m **7.** \$22,200 **9.** 420 mL of vinegar and 315 mL of oil **11.** 21,000 votes, 17,500 votes, and 17,500 votes

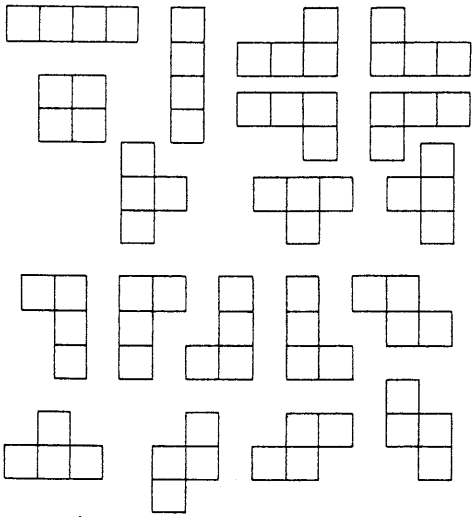
Written Exercises, page 309

1. $x = 9$ **3.** $x = 12$ **5.** $x = 2$ **7.** $x = 8$
9. $a = 2$ **11.** $x = 10$ **13.** $x = 8$
15. $a = 5$ **17.** $x = 2$ **19.** $a = -4$
21. $n = 5$ **23.** $y = -5$ **25.** $x = 3$
27. $m = -1$ **29.** $y = -1$ **31.** $x = 12$
33. $c = 7$

Written Exercises, pages 311–313

1. \$1.55 **3.** \$6.90 **5.** 7 hours **7.** 8 g
9. 21 cm **11.** 8 km **13.** 880 cm **15.** \$495
17. \$18,000

Puzzle Problems, page 313 There are 19 different ways as shown:



Written Exercises, page 315

1. $\frac{1}{12}$ **3.** $\frac{8}{21}$ **5.** $\frac{y^2}{2}$ **7.** $\frac{2}{3}$ **9.** $\frac{2y}{5}$
11. $\frac{1}{2y^2}$ **13.** $6r$ **15.** $\frac{5a^2}{4b}$ **17.** $\frac{x-1}{2}$
19. $\frac{x-4}{2}$ **21.** $5a-5$ **23.** $\frac{a}{ab-b}$
25. $\frac{a+b}{2}$ **27.** $\frac{r}{2}$ **29.** $-\frac{1}{3x+6}$
31. $\frac{x+2}{2}$ **33.** $\frac{2x-10}{x-1}$ **35.** $\frac{x+2}{x+4}$
37. $-7n-7$ **39.** $\frac{m-2}{m+2}$

Written Exercises, pages 317–318

1. $\frac{5}{2}$ **3.** $\frac{6}{5}$ **5.** $\frac{x^2}{12}$ **7.** $\frac{1}{ab}$ **9.** $\frac{21m}{8}$
11. $6b$ **13.** $\frac{ab}{8}$ **15.** 2 **17.** $\frac{a^2+1}{a}$
19. $\frac{2}{y+3}$ **21.** 1 **23.** $\frac{x+4}{2x}$ **25.** $\frac{a-3}{a+3}$
27. $\frac{3y+6}{2y^2-10y+12}$ **29.** $\frac{y-3}{2}$ **31.** $d-5$
33. $\frac{y+2}{y-6}$