Theorems

For all $a, b, c \in \tilde{N}$,

Thm G1

$a = b \Rightarrow a \times c = b \times c$ and $a = b \Rightarrow a + c = b + c$

Thm G2 (Cancellation Theorem)

$a \times c = b \times c \Rightarrow a = b,\ c \neq 0$ and $a + c = b + c \Rightarrow a = b$

Thm F1

$0 \times a = 0$

Thm F2

if $a \times b = 0$, then at least one member of $\{a, b\}$ must be zero.

Thm F3

$a \times (-b) = -(a \times b)$

Thm F4

$(-a)(-b) = ab$

Thm F5

$-(-a) = a$

Thm F6

$-1 \times a = -a$

Theorems Concerning Fractions

For all $a, b, c, d \in \tilde{N}$,

- **Theorem R1**

$$\frac{1}{ab} = \frac{1}{a} \times \frac{1}{b}, \text{ where } ab \neq 0$$

- **Theorem R2**

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}, \text{ where } bd \neq 0$$

- **Theorem R3**

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}, \text{ where } b \neq 0$$

Propositions

For all $a, b \in \tilde{N}$,

- **P1**

$$a = \frac{a}{1}$$

- **P2**

$$\frac{a}{a} = 1, \text{ where } a \neq 0$$

- P3

$$\frac{1}{\frac{b}{a}} = \frac{a}{b}, \quad \text{where } ab \neq 0$$

à **Note that theorem F1 implies that division by *0* is undefined. Since $0 \times a = 0$ for all $a \in \tilde{N}$, there exists no number a such that $0 \times a = 1$. That is, *0* has no multiplicative inverse.**