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6.5 ALGEBRAIC PROBLEMS

Worked Example 12
The total cost of 5 pens and 15 pencils is \$33. If the pencils cost 20 cents each, find the cost of each pen.

Solution:
Let the price of each pen be \$$x$. Then, 5 pens cost \$$(5x)$.
15 pencils cost $15 \times 20¢$, i.e. \$3.
The total cost of the pens and pencils is \$$(5x + 3)$. This cost is given as \$33.
Thus,

$$\begin{aligned} 5x + 3 &= 33 \\ 5x &= 30 \\ x &= \frac{30}{5} \\ &= 6 \end{aligned}$$

Therefore, the price of 1 pen is \$6.

Note: Checking of answers is always beneficial even though it is not required to be written down as part of your answer.

Worked Example 13
The sum of two numbers is 24. Twice the first plus the second is 26. Find the numbers.

Solution:
Let x be the first number. Then, $(24 - x)$ is the second number.
But twice the first plus the second is 26.
Thus,

$$\begin{aligned} 2x + (24 - x) &= 26 \\ 2x + 24 - x &= 26 \\ 2x - x &= 26 - 24 \\ x &= 2 \end{aligned}$$

Also,

$$\begin{aligned} 24 - x &= 24 - 2 \\ &= 22 \end{aligned}$$

Therefore, the numbers are 2 and 22.

Worked Example 14
A boy is 24 years younger than his father. In 2 years' time, the sum of their ages will be 40. How old is the father and how old is the son now?

Solution:
Let the father's age now be x years. Then, the son's age is $(x - 24)$ years.
In two years' time, their ages will be $(x + 2)$ and $(x - 24 + 2)$ years respectively.

Since the sum of their ages will then be 40 years, we have

$$\begin{aligned}(x + 2) + (x - 24 + 2) &= 40\\ x + 2 + x - 24 + 2 &= 40\\ 2x - 20 &= 40\\ 2x &= 60\\ x &= 30\end{aligned}$$

Therefore, the father is 30 years old now and the son is (30 – 24), i.e. 6 years old.

Worked Example 15

A man mixes 5 kg of grade *B* coffee powder with 4 kg of grade *A* coffee powder. The grade A coffee powder costs \$4.50 per kg more than the grade *B* coffee powder. If the cost of the mixture is \$16 per kg, find the price of 1 kg of grade *B* coffee powder.

Solution:

Let \$$x$ be the cost of 1 kg of grade *B* coffee powder. Then, $\$\left(x + 4\frac{1}{2}\right)$ is the cost of 1 kg of grade *A* coffee powder. We have

$$\begin{aligned}5x + 4\left(x + 4\tfrac{1}{2}\right) &= 16 \times 9\\ 5x + 4x + 18 &= 144\\ 9x &= 126\\ x &= 14\end{aligned}$$

Therefore, the price of 1 kg of grade *B* coffee powder is \$14.

Here are some points to remember when solving word problems:

1. Try to understand the problem clearly.
2. Look for the unknown quantities.
3. State clearly what your variable represents and the units used.
4. If necessary, write expressions for other unknown quantities in terms of the variable.
5. Write an equation which states the fact of the problem.
6. Use a sequence of equivalent equations to solve that equation.
7. Check the answers by testing them with the conditions as stated in the problem.