

On The Transition in Mathematics from High School to Michigan State University

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Draft – May 1, 2006

Abstract. We examine relationships between senior-year high school mathematics courses and entry-level mathematics courses at Michigan State University (MSU) using a database of nearly 3000 students from 34 high schools. The results show that high school courses vary tremendously, both between schools and within schools, in terms of preparing students for mathematics at MSU. They also show that high school calculus courses, both AP and non-AP, are generally successful at preparing students for college mathematics, while AP Statistics courses are not, nor are many second and third tier senior high school courses. Finally, the results show that students with no senior-year math usually place into low-level college math courses and do poorly in those.

This study involved 34 high schools and just under 3000 of their students who entered Michigan State University (MSU) over the four-year period 1996 – 1999. Our interest is in exploring the relationships between senior-year high school mathematics courses and entry-level mathematics courses at MSU.

Our interest in this issue came about as a result of various anecdotal reports from colleagues who teach mathematics courses at MSU. For example, among freshmen who placed into precalculus vs. technical calculus at MSU, while there appeared to be relatively small differences in American College Testing (ACT) scores and small differences in high school grade point averages (GPAs), anecdotal evidence by our instructors suggested tremendous differences in how much the students can be pushed, in their willingness to work on challenging problems, and how hard they work on their own. We became interested in conducting a more systematic inquiry into students' senior-year mathematics preparation, with a particular focus on factors that distinguish students who place into the five different tiers of mathematics courses that freshmen take at MSU. More explicitly, our Main Question is:

Can we determine the efficacy of AP calculus, non-AP calculus, AP statistics, reform curricula, or other curricula in terms of preparing students for math at MSU?

This is an important question for colleges and universities to consider. Approximately 70% of high school graduates actually go to college within two years of graduating. (Education Trust, 1999) Mathematics preparation is at the core of college success, irrespective of major. This is the conclusion of a thorough 10-year study examining factors that contribute most to bachelor's degree completion, done by Clifford

Adelman, a senior research analyst at the US Department of Education (Adelman, 1999). Selected findings include:

"Of all pre-college curricula, the highest level of mathematics one studies in secondary school has the strongest continuing influence on bachelor's degree completion. Finishing a course beyond the level of Algebra 2 (for example, trigonometry or pre-calculus) more than doubles the odds that a student who enters postsecondary education will complete a bachelor's degree." (See the Executive Summary – Selected Findings.)

"Of all the components of curriculum intensity and quality, none has such an obvious and powerful relationship to ultimate completion of degrees as the highest level of mathematics one studies in high school." (See Chapter 1 - HIGHMATH)

Complementing this concern for how high school can prepare students for college is the more general issue of the mathematical preparation of American high school graduates. For example, a recent National Assessment of Educational Progress (NEAP) Report (Braswell, et al., 2000) concluded only 17 percent of US twelfth graders were proficient at mathematics. The Third International Mathematics and Science Study (TIMSS) and its follow-ups show the mathematical preparation of US students is woefully inadequate as compared to most other countries. (Takahira, et al., 1998; Mullis, et al., 2004) Related studies suggest that US high schools are offering a wide assortment of courses that lack the focus and coherence found in many foreign curricula. (Schmidt, 2003) Furthermore, there appears to be a disconnect between the mathematics expectations that students encounter in K-12 education and those that they encounter in college. This is among the conclusions of the recent National Research Council (NRC) report on *Evaluation Curricular Effectiveness* (NRC, 2004, p. 161) and of Stanford University's Bridge Project. (Kirst, 2004 and Venezia, et al., 2003) Among other things, these reports call for studies to articulate specifics about the high school mathematics needed for success in college and university courses. The present study hopes to begin to address some of these issues.

Research Questions

The Main Question of this study, mentioned above, can be broken down into three main research questions.

1. How much do high school senior mathematics courses vary in terms of kinds, effectiveness, and description, both from school to school and internally within a given school?
2. What can be said about students who place into different tiers of entry-level mathematics courses at MSU, with particular interest in those students who place into remedial mathematics?
3. (Most important) How effective are given mathematics curricula across schools in terms of placement and grades in the first math courses taken at MSU?

This paper is organized as follows: After the preliminary material, the results of the study are presented in three parts corresponding to the three research questions. In the first part, data from representative high schools are presented that illustrate patterns that can be observed in the total data. They illustrate how tremendously high schools math courses vary – in number, in content, and in how well they prepare students for mathematics at MSU. We observe there is a rough relationship between ACT math scores and MSU math placement test scores. This relationship is used to suggest when there is “value added” (defined later) by specific high school math courses.

In the second section we investigate the backgrounds of students who place into different tiers of freshman math at MSU, with particular attention to remedial mathematics. In the third part various senior-level high school math courses and curricula are examined as to their effectiveness in preparing students for math at MSU.

We now turn to the preliminary material.

Preliminary Material

Literature Review

First and foremost, there is an excellent 10-year study examining factors that contribute most to bachelor’s degree completion, done by Clifford Adelman, a senior research analyst at the US Department of Education (Adelman, 1999). This study with selected findings is mentioned above.

Adelman’s study also has interesting findings about minority students, low SESⁱ students, AP students, and much more. The present study complements Adelman’s study in that it is more narrowly focused on factors that effect success in mathematics (at Michigan State University).

There are numerous studies that examined the effectiveness of AP calculus, some more than twenty years old (Dickey, 1986; Austin, 1975; Chamberlain et al, 1968; Paul, 1970; Rusch, 1968) and some quite recent (Dodd, 2002; Morgan, 1998, 2000). Generally these studies have found that AP calculus is effective in preparing students for college mathematics classes. However, these results have been obtained by comparing AP students with students who took calculus 1 in college. Missing from these studies is a comparison of students who took AP calculus in high school with those who took non-AP calculus. There is anecdotal evidence that students who take non-AP calculus fair poorly in college. We address this missing piece in the present work.ⁱⁱ

There are also studies and reports addressing articulation between high school and college (Cornelius, 1974; Blanchard, 1971; Fawcett, 1981; Webb, 1997). Surprisingly, missing from this literature are studies that examine the effects of high school grades on college grades, as is done here.

*Method*ⁱⁱⁱ

Forty-five high schools were invited to participate in this study, schools from various types of areas in central Michigan that sent at least 10 students to MSU in 1999.^{iv} Thirty-four schools agreed, and they received a list of all students who graduated from

their school in the years 1996 – 1999 and came to MSU, a total of $N = 2961$ students. These schools returned information on the mathematics courses taken and grades received by those students in their senior year of high school. These data were matched with similar data on those students' MSU mathematics courses, with ACT scores, and with MSU's math Placement Exam scores. Partially as an inducement to participate in the study, each school was sent a confidential report on the performance of their students at MSU, data that they were delighted to receive and were otherwise difficult to obtain.

The high schools (32 public, 2 parochial) represented a cross section of sizes and economic backgrounds of students, as indicated in Table 0.

Table 0.

Demographics of the 32 public high schools in the study.

Size			Free or Reduced Lunch		
# of students/school	# schools	%	% receiving	# of schools	% of schools
400 – 999	9 schools	28%	1% - 6%	16	50%
1000 – 1599	15 schools	47%	7% - 19%	9	28%
1600 – 2500	8 schools	25%	20% - 50%	7	22%

There is a separate spin-off study on Core-Plus students (Hirsch et al, 2003) who matriculated at MSU during this time period. See (Hill & Parker, 2006).

Freshmen Math Courses at MSU and math placement

For this study we have grouped the freshman-level MSU mathematics courses into five tiers. The following discussion describes those tiers and gives background information about the MSU mathematics program.

Nearly 7500 freshmen enter MSU each fall. MSU uses a Mathematics Placement Exam^v designed by the MSU Mathematics Department in an attempt to place students into mathematics courses that are at a level appropriate for them. As is typical for such tests at most large universities in the US, this test focuses mainly on algebraic skills. The exam is evaluated each year (and revised when appropriate) using several different standard statistical checks of the correlation between Placement Exam scores and the grades students achieve in their freshman courses. These internal Mathematics Department studies show that the Placement Exam generally directs students into courses where they can succeed.

Here is a brief description of the five tiers, including the Placement Exam score (out of a maximum score of 28) needed to place into the courses in that tier. Note that lower-numbered tier courses are more mathematically advanced. These are all one-semester courses.

1. Tier 1: Technical calculus or a higher-level course. Admission into these courses requires a score of 19 on the placement Exam, an ACT math score of 28, a SAT math score of 640, or a passing score (≥ 4) on the AP calculus exam.
2. Tier 2: Five different courses: (a) business/biological science calculus, (b) trigonometry, (c) mathematics for elementary education students, (d) any statistics course, and (e) a (relatively new) liberal arts math course. Admission into these courses requires a Placement Exam score of 15 (except the trigonometry course, which also requires credit for college algebra).
3. Tier 3: Precalculus. A traditional precalculus course designed to prepare students for technical calculus, although about half of the students are enrolled for other purposes. Admission into this course requires a Placement Exam score of 12.
4. Tier 4: A traditional “College algebra” course for three credits, and a five-credit course on finite math and algebra. A placement score of 10 or higher places students into this tier.
5. Tier 5: Intermediate algebra, which is essentially a remedial math course for college students. It is a three-credit course that does not earn credit towards graduation but it does count towards a student’s GPA.

As a requirement for graduation, all MSU students must pass one mathematics course beyond the level of college algebra (Tier 4). Many large universities have similar graduation requirements. This requirement can be met either by scoring at least 19 on the (proctored version of the) Placement Exam or by receiving credit in any mathematics or statistics course except for those in Tiers 4 and 5.

For this study, students were assigned to the tier of the first mathematics course they took at MSU. Students who took no MSU mathematics course were placed either into the tier of the highest-level course for which they had received transfer or AP credit, or into a separate group labeled “None” if they had no such credit. MSU has a 4.0 grading scale.

Results

The results of the study are organized to respond to the three research questions mentioned at the beginning of this paper.

Research question 1. How much do high school senior mathematics courses vary in terms of kinds, effectiveness, and description, both from school to school and internally within a given school?

Some general comments are given first, followed by some individual school’s data to illustrate several relationships.

The high school curricula of the schools studied is *de facto* broken up into different levels or strata. The current strata are informal and can be difficult to classify, but they are obviously there when one examines the level of the mathematics in the different courses available to seniors. (Typically, AP Calc. is the top stratum, precalculus the second, consumers' math the third, etc.).

In all of the schools in the study, the top-level course (often AP Calculus) does very well (whether the school is urban, suburban, or rural). By this we mean students who do well in the course and come to MSU do well in whatever math course they take at MSU. (Of the students who got at least a C+ in the top-level course and earned a grade in a math course at MSU, their average math grade was 3.06 and 81% placed into a Tier 1 or 2 course at MSU.) Furthermore, there was often "Value added" (to be defined below) in these courses.

The most interesting (perhaps disturbing) result of this section is how much courses below the top stratum vary from school to school and from the top stratum of the same school. There are some schools where students who did well in second and third tier math courses, came to MSU and did well in whatever math course where they placed. (From five of the better schools, of the 143 students who got at least a C+ in the second or third level courses and earned a grade in a math course at MSU, their average first MSU math grade was 3.01 and 52.4% placed into a Tier 1 or 2 course at MSU.) However, at other schools, students who did well in math courses below the top tier placed low and did poorly in MSU math courses. (From ten of the lowest-performing schools, of the 244 students who got at least a C+ in the second or third level courses and earned a grade in a math course at MSU, their average math grade was 2.31 and only 27.9% placed into a Tier 1 or 2 course at MSU.) Courses labeled "precalculus" (and are not that school's top course) can be problematic.^{vi} There are schools where almost no students from their senior-level precalculus course ever make it into technical calculus at MSU. (From five schools, the ratios of (# precalculus students passing Calc. 1)/(# precalculus students) are 1/7, 1/3, 3/12, 1/15, 4/57.)

Here are other interesting differences among high schools.

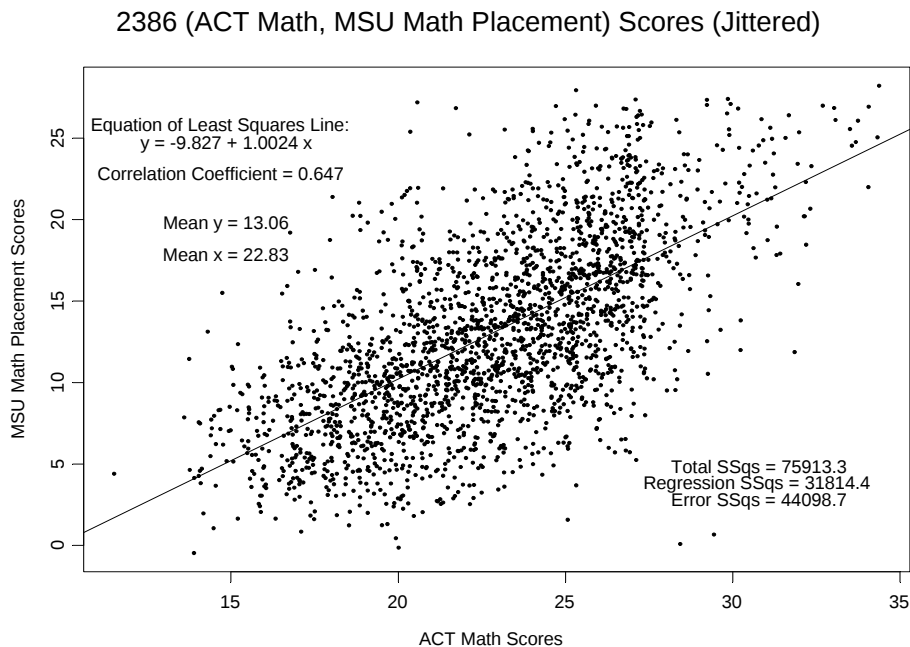
1. The variation in senior-level high school math courses is considerably more than was anticipated. Altogether, there are twenty-nine different courses offered by the high schools we studied. They included AP Calculus, non-AP calculus, precalculus, honors precalculus, trigonometry, FST^{vii}, probability, AP statistics, statistics, finite math, integrated math, consumers' math, etc.
2. One high school offers two different math courses to its seniors; another, ten.
3. The percentage of students who came to MSU after taking no math their senior year varied tremendously from school to school (and generally such students placed into a very low math course at MSU). The median percentage was 18%; the five lowest were 0.7%, 3.0%, 8.8%, 9.4%, 11.5% and the five highest were 32.0%, 32.0%, 33.3%, 34.3%, 40.5%. The performance of such students is detailed under Research Question 3.

Before looking at individual school data, we examine the correlation between ACT scores and MSU's Math Placement scores. This is used to define "value added" in senior-level high school math courses.

ACT Math versus MSU Math Placement. The ACT test is generally taken at the end of a student's junior year in high school, and MSU's mathematics Placement Exam is generally taken at the end of the senior year. Thus it is reasonable to look for a relationship between the two scores that might be used to evaluate senior-level high school math courses.

In this study, there were $n = 2386$ students who had both an ACT math score and a MSU math placement score. The jittered data points, together with the regression line, are plotted in Figure 1, where "jittered" means the points have been perturbed by small random amounts.^{viii}

Figure 1.



The correlation coefficient, 0.647, says the regression line, $y = 1.0024x - 9.827$, yields a reasonable, though rough, relationship. Since the relationship is rough, we shall shorten and rewrite the regression line's equation as $P = A - 9.8$ (for Placement exam score = ACT math score - 9.8).^{ix}

Value Added. The regression line in Figure 1 is a kind of “average” of how students score on the Math Placement Exam as compared to their ACT Math score. What is interesting is that most of the students in some courses at some schools do better than this “average.”

When the increase is statistically significant, we say *these courses give value added* to their students.

This will be illustrated in what follows. Please note that this is a statistical relationship. It suggests, but does not prove, a causal relationship.

An issue regarding the grouping of high school grades. In Tables 1 – 4, data of students in various math courses from various high schools are grouped by high school grades. Since the n 's are small, we are forced to use the natural breaks in the data, which vary from table to table. The result is that the breaks are not uniform from table to table, but the story the data tell is quite apparent.

Specific High Schools. We now turn to the data of a few specific high schools that illustrate the points made at the beginning of this subsection. To preserve confidentiality, these schools were coded as HS1, HS2, etc.

HS1 (the first school to supply us with data) is typical of schools where all math courses “do well.” It is a good-sized suburban school, which sends about 40 students a year to MSU. Its recent history is interesting. In the mid 90's it introduced AP Calculus, and used FST (Senk, 1998) for precalculus. They soon found that lowest AP Calc students were slowing down the class and that the highest FST students were not being well-served. In 1998, they introduced a course based on *Precalculus and Discrete Mathematics* (Peressini, 1998, hereafter PDM) to fit in the middle, putting the two groups of students just described into it. You will see below that this somewhat worked well, but there are too few students for definitive conclusions. For '98 & '99, the students from HS1 who came to MSU either took one of these three courses (AP Calc., PDM, or FST) or took no math at all (a surprising 40.5%).

Table 1 contains the data of those HS1 students who took a math course their senior year and came to MSU. Here is what to look for in the table.

1. The AP Calculus students did typically well. There were 43 such students. Two received AP credit and took no more math at MSU; the data for the other 41 are in Table 1.
2. Concentrating on the two columns that contain ACTM scores and MSU Math scores, those AP Calculus students who earned a B or better had a nice increase in MSU math placement scores over the predicted score using the relationship $P = A - 9.8$. There were only 13 AP Calculus students who earned a B or better and took both the ACT and MSU Placement tests. These students earned slightly more than six points higher on the MSU math placement than predicted by the relationship $P = A - 9.8$, and this is statistically significant ($p < .05$). This illustrates how we defined, *value*

added, above. For the remaining AP Calculus students, there was an average increase of 3.6, but this is not statistically significant.

3. The number of PDM students is quite small. There is a slight (1.8), but statistically insignificant, improvement of MSU Placement Exam scores over those predicted from the ACTM scores for students who earned at least a C⁺ using the correspondence $P = A - 9.8$.
4. For the students who earned an A- or better in FST, there is a statistically significant ($p < .05$) increase in the Math Placement score (4.5) over the score predicted by the ACTM scores. (These students generally did well in math at MSU.) Thus for these students there is “value added” by their FST course. For the remainder of the FST students, there was a slight (1.1), but statistically insignificant, increase in the Math Placement score over the score predicted by the ACTM scores.

Table 1.

Summary of HS1's data, senior-year math courses, grouped by grades both semesters.

HS Grades ^a	n	Medians		n						
		ACTM	MSU Math	Calc 2	Calc 1	Tier 2	Tier 3	Tier 4	Tier 5	Other
≥ A- in AP Calc	10	29	22	6	4	0	0	0	0	0
≥ B in AP Calc	11	28	23	3	5	3	0	0	0	0
Other in AP Calc	20	25	18	0	5	7	4	4	0	0
≥ C+ in PDM ^b	7	26	18	0	1	3	0	2	0	1
Other in PDM	4	24	10	0	0	0	1	1	1	1
≥ A- in FST	6	24	19	0	3	0	1	0	0	2
≥ B in FST	19	21	13	0	1	2	6	8	2	0
Other in FST	35	21	12	0	3	5	12	11	1	0

^a The data of a student who earned an A one semester and a B the other semester is included in the “≥ B” category. Students who earned ≥ A- both semesters are not included in those who earned ≥ B, etc. These conventions will be used throughout the paper. ^b PDM was offered in 1998-9 only.

HS5 is typical of schools where AP calculus “did well,” but its lower-level courses did not. It is a suburban school which sends about 27 students a year to MSU. Table 2 contains the data of those HS5 students who took a math course their senior year and came to MSU. Here is what to look for in the table.

1. The AP Calculus students did typically well. For the students with both ACTM and MSU Math scores, the increase in MSU math placement scores over the predicted score using the relationship $P = A - 9.8$ is only marginally significant ($p < 0.10$).
2. For precalculus scores using the correspondence $P = A - 9.8$ there is a falloff (not statistically significant) from predicted scores for HS5 in Table 2 (instead of the increases for HS1 in Table 1).

- There were eleven students who took AP Statistics their senior year (1999 only). Of these, four had high ACT scores before going into AP Stat (and did well there), they placed into Tier 1 and 2 courses at MSU, and they did very well. The seven others placed into Tiers 4 or 5, and there was a fall off in their MSU Math Placement scores from those predicted by the ACTM scores and the rule $P = A - 9.8$ (quite noticeable but not statistically significant). See the comments later on statistics courses in general.

In addition, there were five HS5 students (not included in Table 2) who took Algebra 2 their senior year, all in the first two years of the study. Their placements were low and their grades below average.

Table 2.

Summary of HS5's data, senior-year math courses, grouped by grades both semesters.

HS Grades	<i>n</i>	Medians		<i>n</i>						
		ACTM	MSU Math	Calc 2	Calc 1	Tier 2	Tier 3	Tier 4	Tier 5	Other
≥ A- in AP Calc	8	29	22	3	3	0	0	0	0	2
≥ B in AP Calc	3	31	-- ^a	3	0	0	0	0	0	0
Other in AP Calc	7	26	18	2	2	0	1	1	0	1
≥ B+ in Precalc	6	25	14.5	0	0	0	4	2	0	0
Other in Precalc.	38	24	13	0	3	3	10	13	4	5
≥ B in AP Stat ^b	6	26	12.5	0	1	2	0	3	0	0
Other in AP Stat	5	23	8	0	0	1	0	2	2	0

^a There were no students in this category who took the MSU Math Placement test. ^b HS5 started offering AP Statistics in the last year of the study.

HS 17 is typical of schools where students in “business math” courses come to MSU. A medium-sized school that sends about 33 students/year to MSU, it offers three courses to seniors, AP calculus, trigonometry/precalculus, and consumer's math. Its data are in Table 3.

Notice: 1. The (statistically significant, $p < .05$) “value added” in AP Calculus (using $P = A - 9.8$), 2. The losses (not statistically significant) in precalculus and consumer's math, and 3. The big loss (statistically significant, $p < .05$) in “None.” In Consumer's math, there were no grades given $< C$. Also notice that the distribution of placement was better for those students who took no math than for students coming out of Consumer's Math; the grades earned were slightly better, too.

Table 3.

Summary of HS17's data, senior-year math courses, grouped by grades both semesters.

HS Grades	<i>n</i>	Medians		<i>n</i>					
		ACTM	MSU Math	Tier 1	Tier 2	Tier 3	Tier 4	Tier 5	None
≥ C in AP Calc	10	27.5	21	8	0	0	0	0	2
≥ B in Precalc	17	25	14	2	3	4	7	0	1
≥ C in Precalc	17	22	12	0	2	4	7	2	2
≥ B in ConMth	9	19	9	0	0	0	2	7	0
≥ C in ConMth	7	16	5.5	0	0	0	1	5	1
None	10	21	6	0	0	1	3	3	3

HS25 is typical of the (six out of 34) schools in the study that did not offer AP Calculus. It is a small school in a rural suburb of Lansing that sends about 22 students a year to MSU. It offered only two math classes to seniors, precalculus and a FST course. Its precalculus course (for emphasis, its top course) did almost spectacularly well for its students. Using the rule $P = A - 9.8$, HS25's precalculus students' medians were a full two points above the predicted values. (This is statistically significant for the students with grades $\geq B+$, $p < .05$; not for the others.) The grades earned in MSU math courses are quite reasonable, also.

Notice that the FST data tell a different story. The median MSU Math Placement scores are slightly below those predicted by the rule $P = A - 9.8$. This provides another example of a school where the students in the top tier math course have significant "value added," but the students in the second tier do not.

However, the main point is that the HS25 data illustrates that students who do well in the top tier math course can do well in math at MSU even though that top course is not AP Calculus. The data are in Table 4.

Table 4

Summary of HS25's data, senior-year math courses, grouped by grades both semesters.

HS Grades	<i>n</i>	Medians		<i>n</i>						
		ACTM	MSU Math	Calc 2	Calc 1	Tier 2	Tier 3	Tier 4	Tier 5	Other
≥ B+ in Precalc	6	26.5	23	0	4	1	0	0	0	1
Other in Precalc	16	26.5	19	0	4	5	4	2	0	1
≥ B+ in FST	9	24	14	0	0	3	4	2	0	0
Other in FST	30	22	12	0	1	2	6	14	6	1

Urban Schools. Outside of the Detroit region, Michigan has several small metropolitan areas, more or less isolated from one another. We gathered data from high schools in several of these areas to see if we could see any patterns. Unfortunately there are several confounding issues (such as better students attending local community colleges or math/science teaching centers) for us to discern significant patterns. However, there were two things we noticed.

1. These schools had a higher percentage of students with low ACT math scores ($25.7\% \leq 19$, out of $n = 514$) than the rest of the schools ($15.0\% \leq 19$, out of $n = 2317$).
2. At most of these “urban” schools outside of the Detroit area, their top students in calculus or AP calculus courses fared about as well as comparable students in suburban schools (including “value added” in these courses).

Thus these urban schools had more weak students than the suburban schools, but they still seemed to serve their top students well.

Research question 2. What can be said about students who place into different tiers of entry-level mathematics courses at MSU, with particular interest in those students who place into remedial mathematics?

The main motivation for this section was to determine the background of students who placed into remedial math (intermediate algebra) and college algebra, i.e., into Tiers 5 and 4. (For completeness we shall do this also for Tiers 3, 2, and 1.) As stated previously, MSU has a graduation requirement that all students must pass a mathematics or statistics class beyond college algebra. Historically, about 25% of incoming freshmen place into intermediate algebra, 15% place into college algebra, so altogether about 40% of incoming freshmen are required to take a second or third math or statistics course to graduate, regardless of major. (This is sometimes viewed as a failing of the university’s placement process.) So the question is, who are these students, are they being unfairly penalized, or are there factors in their backgrounds that explain their placement? We first examine Tiers 5 and 4.

Tier 5 – Remedial Math. 550, or about 18%, of the students in this study started out in remedial math, Tier 5 (and earned an average grade of 2.28). In addition, there were another 25 who took no math at MSU but who would have started with this course if they had taken math.^x Thus about 20% of the students in the study placed into remedial math. Here are the findings:

Over 75% of the remedial students have a factor in their background that explains their placement.

In detail (and students were not double-counted):

1. 194 or about 35.3% of them took no math their senior year in high school.
2. 88 or about 16% of them took only one course their senior year (and this does not count students from block-scheduling high schools).
3. 101 or about 18.4% of them took low-level or non algebraically-demanding math their senior year.
4. 33 or about 6% of them took a potentially demanding course but received low grades ($< C$).

For the remainder of these students, the reasons are just not clear and require deeper investigations to answer. (Indeed two students came from AP Calculus!)

Tier 4 – College algebra and Finite Math. 689, or about 24%, of the students started with a Tier 4 course, earning an average grade of 2.2. As compared with Tier 5, above, 407 (or about 60%) had no math or a weak senior-year high school math background. It is a little surprising that 199 (30%) had calculus or precalculus mostly with grades of C or higher. The rest came from a scattering of courses.

We now summarize the results for all tiers in Table 5 and then make a few comments.

Table 5
Summary of results of students by Tier.

Tier	n	% ^a	Average Grade ^b	HS Background Comments
5	550	18%	2.3	75% from a weak background
4	689	24%	2.2	60% from a weak background; 30% had $\geq$ C in calc. or precalc.
3	435	15%	2.3	38% had $\geq$ C in calc. or precalc; most had $\geq$ C. ^c
2	410	14%	2.8	50% had $\geq$ C in calc. or precalc; most had $\geq$ C. ^c
1	604	21%	2.8	79% had calc. or precalc, $\geq$ B. ^c

^a Of all students in the study. ^b In the MSU course. ^c In whatever senior-level math course they had in high school.

There are two things to notice in Table 5. First, as one moves from Tier 4 to Tier 1, the backgrounds of the students are getting stronger (to be expected). Second, there is a surprisingly high percentage of students in each of Tiers 4 – 2 that had precalculus or calculus; thus these students repeated at MSU the math they had seen in high school.

Research question 3. How effective are given mathematics curricula across schools in terms of placement and grades in the first math courses taken at MSU?

In this section, we turn to the third and final research question. We examine the placement and grade averages of students coming out of various senior year curricula. We start with Table 6 that summarizes the placement of students who took six of the more common senior-year high school math curricula (including “None”). A discussion of the students from each category follows. Note that the category “Statistics” is broken up into three subcategories: (a) statistics one semester and an algebraically demanding course the other semester, (b) AP statistics (for two semesters), and (c) non-AP statistics for one semester with either no math or another statistics course the other semester. These three subcategories require special attention. After this discussion, there is one special grouping of curricula involving trigonometry that is discussed separately.

Table 6

Placement of Students into the Tiers of first-semester Mathematics Courses at MSU, by Senior-Year High School Math Curriculum

Curriculum	<i>n</i>	Tier 1	Tier 2	Tier 3	Tiers 4 & 5	None
None	564	5%	9%	9%	70%	6%
AP Calc	491	66% ^a	15%	11%	5%	3% ^b
Calc, not AP	117	51%	14%	20%	13%	3%
Precalculus	571	18%	23%	22%	35%	3%
Discrete Math	61	10%	30%	18%	43%	0%
Statistics						
1 Sem.+ an algebr. demanding course	33	48%	21%	21%	9%	0%
AP Statistics	26	12%	35%	0%	54%	0%
1 Sem.+ None or 2 Semesters	38	11%	18%	8 %	58%	5%

^aOf this 66%, 8% received AP credit and took no more math, and 27% received AP credit and started in Calc. 2 or 3. Of the remaining 31%, we cannot tell how many did not take the AP exam and how many took it but did not receive credit. Many of the high schools did not distinguish between AB and BC calculus students, so our data combines these two levels of AP students. ^bSeven of the 15 students in this category satisfied MSU's math requirement by getting at least a 19 on the *proctored version* of the math placement exam.

The categories in Table 6 are now discussed, beginning with “None” to emphasize the consequences of students not taking math their senior year.

None. For $n = 564$, or about 19% of the students in our study, MSU has no record of their taking math their senior year in high school. It is likely a few of these students took math at a local college or a math/science teaching center and this was not indicated in the high school records. For example, twenty of these students started with Calc 1; eight, Calc 2; four, Calc 3. Four more had transfer credit for Calc 1 and took no more math at MSU.

The important thing to notice is that over 75% of the students who took no math their senior year in high school either placed into remedial math, into Tier 4, or took no math at all at MSU. As we noted above, the percentage of students who take no senior-year math varies tremendously from school to school. Table 6 shows they are at an extreme disadvantage when they go to college. These data confirm studies such as (Waits & Demana, 1988) that show that three years of high school math is not enough for success at major colleges and universities.

AP Calculus. Just under 17% of the students ($n = 491$) in the study took AP Calculus their senior year, and generally they did quite well in math at MSU. We do know many students take the AP calculus course intending not to take the AP exam, and we have no data as to how many students actually did or did not take the exam.

Interestingly, the AP calculus students who placed into precalculus averaged only 2.6, whereas those who placed into Tiers 1 and 2 averaged 3.1. Overall, AP Calculus is a

very structured program, and it prepares students well for a traditional math program found at schools like MSU.

Calculus, not AP. There were 117 students from ten schools that took a calculus course that was not AP Calculus. Generally these students did well in math at MSU. This is somewhat surprising, since it is folklore in the math educational community that students should take either an AP calculus-type course or no calculus. See, for example, (Steen, 1987, pp. 218-9).^{xi}

A few schools had both AP and non AP calculus. At some schools, a few of the non-AP Calculus students who did very well were encouraged to take the AP exam. Also, a few students took this course in the fall and then took calculus for college credit, likely at a nearby college. Two got AP credit and took no more math, and a few started with Calc. 2 or Calc. 3, about half doing very well and half doing very poorly.

Precalculus. It was noted above that precalculus courses varied tremendously from school to school, and this is reflected in the Table 6 data. There were 571 students from 24 high schools in a course called precalculus or PDM (Precalculus/Discrete Math).^{xii} Comments on the precalculus data are given in the conclusion.

Discrete Math. There were 61 students from three high schools in courses called Discrete Math or Discrete Math/Precalculus. Of course, their numbers are too small for definitive conclusions, but it is interesting to compare their results with students in the other categories in Table 6. Discrete math seems to do a little better than precalculus, a surprising result. It also does a little better than pure statistics, about the same as AP statistics and more than a little worse than statistics plus an algebraically demanding course. Clearly, a more thorough study is required to unravel the causes.

Statistics. Many students in the study saw some kind of statistics in high school. In some curricula, it is integrated into other math courses, such as in FST (Senk, et al, 1998) where the “S” is for Statistics. In Core-Plus (Hirsch, et al, 2003) statistics is intermingled with all of the other topics. In this study, there were 97 students from eight schools that had a “stand alone” statistics course; two of these were AP statistics. These fell into three subcategories, as described in the paragraph before Table 6.

It is very striking how differently the students from the three subcategories did in math at MSU. The 33 students who had one semester of statistics together with one semester of an algebraically demanding course did well at MSU. The others did only marginally better than the students who took no math at all their senior year.

These numbers are very small, and no one should attempt to draw any definitive conclusions from them.

This completes the discussion of the curricula in Table 6. There is one special grouping of curricula remaining to be discussed.

Trigonometry. In the *Literature Review*, we quoted Clifford Adelman (1999) saying: “Finishing a course beyond the level of Algebra 2 (for example, trigonometry or pre-calculus) more than doubles the odds that a student who enters postsecondary education will complete a bachelor's degree.” This section expands on Adelman’s

comment, showing that to do well in math at MSU a student needs more than just trigonometry.

There were 120 students from twelve schools who had a course their senior year called “Trigonometry.” Mostly, this was a one-semester course, followed by one of three options: (a) an algebraically demanding course (usually called precalculus or intro to calculus), (b) a statistics course, or (c) nothing. Table 7 shows the remarkably large role that second-semester course played in the placement into mathematics at MSU. Part of the story is the math background, as measured by ACTM and MSU math placement scores. That is why these data are included in Table 7.

Table 7

Students Who Took Trigonometry Fall Semester of their Senior Year.

Spring		Standardized Tests			Tier of First-Semester MSU Math Course				
Sem. Course	<i>n</i>	ACTE	ACTM	MSU	Tier 1	Tier 2	Tier 3	Tiers 4 & 5	None
Algebraically									
Demanding	39	23.6	24.1	14.7	18%	23%	26%	31%	3%
Statistics	35	22.6	21.9	11.1	5%	11%	23%	57%	3%
Nothing	46	22.8	21.3	9.8	4%	4%	2%	89%	0%

Note. ACTE means average ACT English score, ACTM means average ACT Math score, and MSU means the average MSU Math Placement score.

Note that the ACT English average scores for the three groups are very similar. Based on this one might assume that the students in the three categories are comparable. But thereafter this table has some dramatic differences.

1. The “algebraically demanding” students came in with better math scores than the other two groups, and their result are quite a bit better. Indeed, the results are better than the results of the precalculus students in Table 6.
2. 89% of the “Nothing” students placed into Tiers 4 & 5; this is worse than what is reported in Table 6 for students who took no math at all their senior year!
3. The data for trig plus statistics is comparable to the data in Table 6 of the students who took two semesters of statistics. It is only slightly better than the data for trig + nothing.

These data call out for a more thorough study of what is going on in these courses and the placement of students into them.

Limitations of the Methodology

This is a descriptive study. There are just a few firm conclusions, but there are several very interesting relationships suggested by the data. Thus, this study points the way for several follow-up, in-depth investigations.

This study examines high school data from only students’ senior year and from only 34 high schools. To understand fully how high school mathematics varies will require student data from all four years of high school and information about placement into different tracks in high school. It will also require data from many more universities

and from many more high schools, as some curricula (such as AP statistics) were taught at only a very few high schools in the study.

Summary and Conclusion

Since colonial times, for better or worse, local education has been under local control. Hence it is not surprising that school curriculum varies from system to system, but it is surprising that it varies as much as it does both in content and in effectiveness. Here is a summary of some of the results of this study, where:

works well has the very restrictive meaning of “students who got good grades in this high school math course also got good grades in the math course where they were placed at MSU.”

For emphasis, most of the following statements are preliminary and will require full blown studies before definitive conclusions can be drawn.

1. AP Calculus works well, as does non-AP calculus.
2. Generally, the top course for seniors in most schools works well.
3. Generally, grades in the top courses are meaningful but grades in lower courses are often problematic.
4. There is a real concern about statistics in high school. The data in Table 6 suggest that very serious studies are needed about the effectiveness of pure statistics courses, especially AP Statistics.^{xiii} Statistics is important and it is good for citizens in a democracy to have a feel for what they do and do not say.^{xiv} But high school level statistics requires very little math, perhaps half of Algebra 1^{xv}. The data in Table 6 suggest that a year-long statistics course should not be thought of as an alternative math course giving preparation for college-level math.^{xvi}
5. It is surprising that trigonometry by itself seems not to work well, but when accompanied by an algebraically demanding course outcomes are improved.
6. The data on precalculus courses in Table 6 are most disturbing. It is important to remember that senior-level course called “Precalculus” is mostly a high school’s second or third tier course. However, it is a genuine reason for concern when courses called “Precalculus” place fewer than 20% of their students into technical calculus, the students there have less than a 2.0 average, and 35% of the precalculus students place into college algebra or remedial mathematics. Furthermore, as noted above, the quality of precalculus graduates varies tremendously from school to school. From some schools, a good percentage of precalculus students makes it into calculus and do well; however, at many schools only a very few precalculus students *ever* make it into technical calculus. Of the several things in this paper that call out for further study, this calls out the loudest.
7. More generally courses such as general math or business math do not work well. They were usually designed for students not going on to college, and college-bound students should be discouraged from taking them.

8. Generally, where students place and how well they do there is a function of both the quality of the course and how well they do in the course. This is consistent with Adelman's findings (Adelman, 1998) mentioned in the second section of this paper.
9. Generally, students who place into remedial math took no math their senior year, took an algebraically undemanding course their senior year, or did poorly in whatever course they took.
10. MOST IMPORTANT, students who take no math their senior year are likely to place low and do poorly. This raises serious concerns about the schools that have a high percentage of graduates going on to college with no senior-year math. (Many high schools have a low percentage of such students.) It especially raises concerns that high schools mostly do not require four years of math (many, not even three), and that most colleges and universities do not require four years of math (many, not even three) as an entrance requirement.

This last item raises the question, whose answer is far beyond the scope of this paper: What should be in a senior-level course intended for college-bound students not in the top tier and perhaps turned off to math?

An important result of this study is to demonstrate the need of gathering course and grade data from both secondary and tertiary schools and comparing the information with standardized test scores. While many of the results are surprising and interesting, it is painfully clear that senior-year high school data is not sufficient to answer many of the questions raised by this study. It will be only by looking at data from all four years of high school, perhaps even into junior high school, and from other colleges and universities, that some of these questions might be answered.

The word *articulate* is very interesting. It often means "to pronounce distinctly and clearly," and it can also mean "to unite by means of a joint," as in skeletons and railroad trains. But in education it is often used as:

"To form or fit into a systematically related whole: interrelated systematically: coordinate coherently <the high schools have been *articulated* with the state university>." (Grove, 1981, p. 124.)

Unfortunately, the joint between high schools and universities has arthritis. Current privacy laws make it almost impossible for high schools to find out how well their students do in college. There are a variety of other contributing factors. In any case, the results of TIMSS (Schmidt, 2003; US Dept. of Ed., 2003) suggest there is something seriously wrong with the underlying skeleton of the US educational system. The study presented here is perhaps one small drop of cortisone, illustrating one possible method of articulation between high schools and universities. It is only by the cooperation of high school and university faculty and staff, which must include university mathematicians, that improvement can be brought about.

Acknowledgements. First and foremost, the author would like to thank the faculty and staff of the 34 high schools that participated in the study, people without whose cooperation this study could not have been done. The author would also like to thank Prof. James Stapleton for continuing assistance with the statistics, Prof. Marshall Hestenes for help obtaining the MSU data, Alice Hill who very carefully read several early drafts of this paper, and Profs. Robert Lovell, Thomas Parker, Sharon Senk, and Jon Star who gave many constructive suggestions for the improvement of the manuscript.

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ⁱ Socio-Economic Status

ⁱⁱ Most of these studies are about AB Calculus. Many of the high schools did not distinguish between AB and BC calculus students, so our data combines these two levels of AP students.

ⁱⁱⁱ Much of the material in this and the following subsection comes from (Hill & Parker, 2007, pp. 909-10).

^{iv} This region was chosen because MSU is in central Michigan.

^v A sample exam can be found at <http://www.msu.edu/~mps/mps/sample/>.

^{vi} This makes decision making difficult for the Office of Admissions.

^{vii} *Functions, Statistics, and Trigonometry*. See (Senk, et al, 1998).

^{viii} These data come in integer pairs, and if they were plotted exactly, many points would lie on top of one another. When they are jittered, you see them all and have a better visual sense of the data's distribution.

^{ix} As an example, generally an ACT score of 18 and under indicates a student is at risk, and this corresponds to a Placement Exam score of 9 and under which places the student into remedial math.

^x These students dropped out of MSU without graduating.

^{xi} To be accurate, what Steen and Dossey wrote was that high school calculus courses should be of sufficient quality that students who do well in such courses should be expected to take the AP (or equivalent) exam and place out of Calculus 1.

^{xii} Most of the other schools had a trigonometry course that played this role.

^{xiii} It is regretful that when the National Academies evaluated AP calculus, physics, and chemistry in [National Academies Press (2002)], AP statistics had not been in existence long enough to have been evaluated, also. As of this writing, the College Board has not presented any data evaluating AP statistics, either.

^{xiv} The book by Joel Best (Best, 2001) is an excellent source for this and would make a wonderful statistical supplement for a math course.

^{xv} College-level statistics with a calculus prerequisite uses considerable math.

^{xvi} AP Statistics credit is given only for academically low-level courses, nowhere near the level of AP Chemistry, English, Calculus, etc. One might wonder why an AP course was even devised at such a level, except for those high school students seeking admission to an elite university who want one more AP course on their high school transcript.