Basic trigonometry

Definition

The point (u,v) on the unit circle is located distance x (measured along the circle) from $(1,0)$. The positive direction along the circle is counter clockwise. Define six functions called sine, cosine, tangent, cotangent, secant, cosecant as follows:

$$\sin x = v \qquad \cos x = u$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

for all values of x for which the denominator is not 0.

From the definition, it is clear that $-1 \leq \sin x \leq 1$ and $-1 \leq \cos x \leq 1$.

Identities

It is understood that the domain of each identity is suitably restricted to exclude undefined expressions.

(1) a) $\sin^2 x + \cos^2 x = 1$

b) $\tan^2 x + 1 = \sec^2 x$

c) $1 + \cot^2 x = \csc^2 x$

(2) a) $\sin(x + 2n\pi) = \sin x$

b) $\cos(x + 2n\pi) = \cos x$

c) $\tan(x + n\pi) = \tan x$, where n is an integer.

(3) a) $\sin(-x) = -\sin x$

b) $\cos(-x) = \cos x$

c) $\tan(-x) = -\tan x$

(4) a) $\sin(\pi + x) = -\sin x$ $\qquad$ $\sin(\pi - x) = \sin x$

b) $\cos(\pi + x) = -\cos x$ $\qquad$ $\cos(\pi - x) = -\cos x$

c) $\tan(\pi + x) = \tan x$ $\qquad$ $\tan(\pi - x) = -\tan x$

(5) a) $\cos(\frac{\pi}{2} + x) = -\sin x$ $\qquad$ $\cos(\frac{\pi}{2} - x) = \sin x$

b) $\sin(\frac{\pi}{2} + x) = \cos x$ $\qquad$ $\sin(\frac{\pi}{2} - x) = \cos x$

c) $\tan(\frac{\pi}{2} + x) = -\cot x$ $\qquad$ $\tan(\frac{\pi}{2} - x) = \cot x$

Addition identities

(6) $\cos(x+y) = \cos x \cos y - \sin x \sin y$

(7) $\cos(x-y) = \cos x \cos y + \sin x \sin y$

(8) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

(9) $\sin(x-y) = \sin x \cos y - \cos x \sin y$

(10) $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

(11) $\tan(x-y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

Double angle identities

(12) $\sin 2x = 2 \sin x \cos x$

(13) $\cos 2x = \cos^2 x - \sin^2 x$

(14) $\cos 2x = 2\cos^2 x - 1$

(15) $\cos 2x = 1 - 2\sin^2 x$

(16) $\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

Half angle identities

(17) $\sin^2\left(\dfrac{x}{2}\right) = \dfrac{1}{2}(1 - \cos x)$

(18) $\cos^2\left(\dfrac{x}{2}\right) = \dfrac{1}{2}(1 + \cos x)$

(19) $\tan^2\left(\dfrac{x}{2}\right) = \dfrac{1 - \cos x}{1 + \cos x}$

Product identities

(20) $\cos x \cos y = \dfrac{1}{2}[\cos(x+y) + \cos(x-y)]$

(21) $\sin x \cos y = \dfrac{1}{2}[\sin(x+y) + \sin(x-y)]$

(22) $\sin x \sin y = \dfrac{1}{2}[\cos(x-y) - \cos(x+y)]$

Sum identities

(23) $\sin x + \sin y = 2\sin\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$

(24) $\cos x + \cos y = 2\cos\left(\dfrac{x+y}{2}\right)\cos\left(\dfrac{x-y}{2}\right)$