Example of properly completed and formatted homework

10/13/08

[HH] 14G #8(c-i), 9a, 10a, 11, 12, 13, 14a

J11 pp 91-94 #1-4

HW

P322 | #8 | all matrices are 2x2, I: identity matrix, simplify:

c. $A(A^2-2A+I)$

$= A^3-2A^2+AI$

$= A^3-2A^2+A$ $\{AI = IA = A\}$

d. $A(A^2+A-2I)$

$= A^3+A^2-2AI$

$= A^3+A^2-2A$

e. $(A+B)(C+D)$

$= (A+B)C+(A+B)D$

$= AC+BC+AD+BD$

f. $(A+B)^2$

$= (A+B)(A+B)$

$= (A+B)A+(A+B)B$

$= A^2+BA+AB+B^2$ $\{AA = A^2,\ B\cdot B = B^2\}$

g. $(A+B)(A-B)$

$= (A+B)A+(A+B)(-B)$

$= A^2+BA-AB-B^2$

h $(A+I)^2$

$= (A+I)(A+I)$

$= (A+I)A+(A+I)I$

$= A^2+IA+AI+I^2$

$= A^2+A+A+I$ $\{I^2=I\}$

$= A^2+2A+I$

i. $(3I-B)^2$

$= (3I-B)(3I-B)$

$= (3I-B)3I+(3I-B)(-B)$

$= 9I^2-3BI-3IB+B^2$

$= 9I-3B-3B+B^2$

$= 9I-6B+B^2$

P323 | #9 | a. If $A^2=2A-I$, find A^3 and A^4 in linear form, $kA+lI$

$A^3 = A\times A^2$

$= A(2A-I)$

$= 2A^2-AI$

$= 2(2A-I)-AI$

$= 4A-2I-AI$

$= 4A-2I-A$

$= 3A-2I$

$A^4 = A\times A^3$

$= A(3A-2I)$

$= 3A^2-2AI$

$= 3(2A-I)-2A$

$= 6A-3I-2A$

$= 4A-3I$

#10 | a. If $A^2=I$, simplify:

i. $A(A+2I)$

$= A^2+2AI$

$= I+2A$

ii. $(A-I)^2$

$= (A-I)(A-I)$

$= (A-I)A+(A-I)(-I)$

$= A^2-IA-AI+I^2$

$= I-A-A+I$

$= 2I-2A$

iii. $A(A+3I)^2$

$= A(A+3I)(A+3I)$

$= (A^2+3AI)(A+3I)$

$= (A^2+3A)A+(A^2+3A)3I$

$= A^3+3A^2+3A^2I+9AI$

$= A+3I+3I^2+9A$

$= 10A+6I$

#11) The result "if $ab=0$, then $a=0$ or $b=0$" for real #s does not have an equivalent result for matrices.

a. If $A=\begin{bmatrix}1&0\\0&0\end{bmatrix}$ and $B=\begin{bmatrix}0&0\\0&1\end{bmatrix}$,

find AB.

$AB=\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}=0$

This example provides us with evidence that "If $AB=0$ then $A=0$ or $B=0$" is a false statement.

b. If $A=\begin{bmatrix}\frac{1}{2}&\frac{1}{2}\\\frac{1}{2}&\frac{1}{2}\end{bmatrix}$, determine A^2.

$A^2=\begin{bmatrix}\frac{1}{2}&\frac{1}{2}\\\frac{1}{2}&\frac{1}{2}\end{bmatrix}\begin{bmatrix}\frac{1}{2}&\frac{1}{2}\\\frac{1}{2}&\frac{1}{2}\end{bmatrix}=\begin{bmatrix}\frac{1}{2}&\frac{1}{2}\\\frac{1}{2}&\frac{1}{2}\end{bmatrix}$

c. Comment on the following argument for a 2X2 matrix A:

It is known that $A^2=A$. $\therefore A^2-A=0$

$\therefore A(A-I)=0$

$\therefore A=0$ or $A-I=0$

$\therefore A=0$ or $A=I$

Because of 'a', this is not necessarily true.

d. Find all 2X2 matrices A for which $A^2=A$.

(Hint: Let $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$)

① $I^2=I$ $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

② zero matrix $\begin{bmatrix}0&0\\0&0\end{bmatrix}$

$A^2=\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}a^2+bc&ab+bd\\ac+cd&bc+d^2\end{bmatrix}$

$A^2=A \Leftrightarrow \begin{bmatrix}a^2+bc&ab+bd\\ac+cd&bc+d^2\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}$

$\boxed{c=\frac{a-a^2}{b}}$ ← 1) $a^2+bc=a$

$b=ab+bd$ ← 2) $ab+bd=b$ $\quad b(a+d)=b \quad a+d=1$

3) $ac+cd=c$ $\quad c(a+d)=c \quad a+d=1$

4) $bc+d^2=d$

$a+d=1$

$\boxed{d=1-a}$

$a=1-d$

$=1-1+a$

$\boxed{a=a}$

$b=ab+bd$

$=ab+b(1-a)$

$=ab+b-ab$

$\boxed{b=b}$

③ $\begin{bmatrix}a&b\\\frac{a-a^2}{b}&1-a\end{bmatrix}$