1. What counts as 100%?

For many problems that involve percentage, the key question to ask and answer is "What counts as 100%?"

In Workbook 6A Exercise 22 #2, we are told two facts:
Fact #1. *There are 400 students in a school hall.*
Fact #2. *240 of these students are boys.*
We can easily discover an additional
Fact #3. *160 of these students are girls.*

We illustrate these three facts in Figure 1.

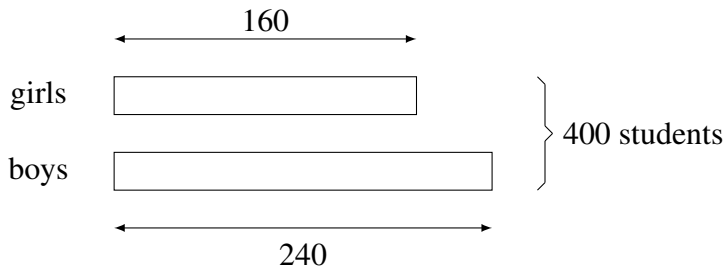

FIGURE 1. Boys and girls at a school.

Given these facts, there are four different questions.

(1) The girls are what percentage of the students?
(2) The boys are what percentage of the students?
(3) The girls are what percentage of the boys?
(4) The boys are what percentage of the girls?

Each question has a different answer. Finding the correct answer to each question depends on knowing what counts as 100%.

Now we answer each question in turn being careful to say what counts as 100%.

The girls are what percentage of the students?

Since the number of girls is being compared to the number of students, the number of students counts as 100%. We want to write 160 out of 400 as a percentage.

$$\begin{aligned} 160 \text{ out of } 400 &= \frac{160}{400} \\ &= \frac{160}{400} \times 100\% \\ &= 40\%. \end{aligned}$$

Therefore, the girls are 40% of the students.

The boys are what percentage of the students?

Since the number of boys is being compared to the number of students, the number of students counts as 100%. We want to write 240 out of 400 as a percentage.

$$\begin{aligned} 240 \text{ out of } 400 &= \frac{240}{400} \\ &= \frac{240}{400} \times 100\% \\ &= 60\%. \end{aligned}$$

Therefore, the boys are 60% of the students.

The girls are what percentage of the boys?

Since the number of girls is being compared to the number of boys, the number of boys counts as 100%. We want to write 160 out of 240 as a percentage.

$$\begin{aligned} 160 \text{ out of } 240 &= \frac{160}{240} \\ &= \frac{160}{240} \times 100\% \\ &= 67\%. \end{aligned}$$

Therefore, the girls are 67% of the boys.

The boys are what percentage of the girls?

Since the number of boys is being compared to the number of girls, the number of girls counts as 100%. We want to write 240 out of 160 as a percentage.

$$\begin{aligned} 240 \text{ out of } 160 &= \frac{240}{160} \\ &= \frac{240}{160} \times 100\% \\ &= 150\%. \end{aligned}$$

Therefore, the boys are 150% of the girls.

2. Percent of what?

Do not forget that a percentage is always a *percentage of something*.

If you do forget this, it is easy to draw incorrect conclusions. For example, we know that the girls are 40% of the students and the boys are 60% of the students. If we ignore the "of the students" phrase, we are tempted to say there are 20% more boys than girls. But that is false.

If we remember the "of the students" phrase, we cannot go wrong.

$$\text{the boys} = 60\% \text{ of the students}$$
$$\text{the girls} = 40\% \text{ of the students}$$

Therefore, 20% more *of the students* are boys than girls.

On the other hand, suppose we ask

How many percent more boys than girls are there?

$$\text{the boys} = 150\% \text{ of the girls}$$
$$\text{the girls} = 100\% \text{ of the girls}$$

Therefore, the boys are 50% more *of the girls* than the girls. This is pretty awkward English, so we just say "There are 50% more boys than girls."