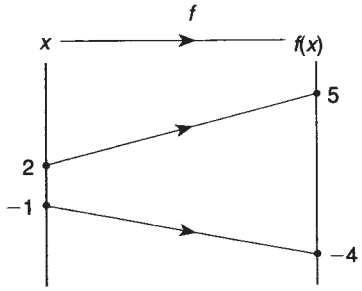
[07-01-01-NAM-11]

The notation for composition used in this book:

fg(x) means f(g(x)).

1. The figure shows part of the mapping

$$f : x \mapsto px + q.$$



Find

(a) the value of p and of q,

(b) the image of 3 under f,

(c) the element whose image is 8.

2. Given that $f : x \mapsto 2 + \frac{a}{x}$, $x \neq 0$, and that $f^{-1}\left(\frac{1}{2}\right) = -2$, find the value of a. Find also the elements which are unchanged under f.

3. Given the functions $f : x \mapsto 2 - x$ and $g : x \mapsto \frac{3}{x}$ (for $x \neq 0$), find in similar form,

(a) f^{-1}, **(b)** ff, **(c)** gg, **(d)** fg, **(e)** gfg.

4. Functions f and g are defined by

$$f : x \mapsto 4x - 3,$$

$$g : x \mapsto 2 - \frac{5}{x}, \quad x \neq 0.$$

Find an expression for the function

(a) ff, **(b)** gf, **(c)** f^{-1}, **(d)** g^{-1}, **(e)** $(fg)^{-1}$.

5. Functions f and g are defined by

$$f : x \mapsto \frac{x - 3}{2},$$

$$g : x \mapsto \frac{3 - x}{x + 1}, \quad x \neq -1.$$

Find

(a) $f^{-1}(2)$, **(b)** $gf(-1)$, **(c)** $fgf(2)$.

Express in similar form, the function

(d) g^{-1}, **(e)** fg, **(f)** gfg.

6. A function f is defined by $f : x \mapsto \frac{x}{1 - x}$, $x \neq 1$.

(a) Show that $f^2 : x \mapsto \frac{x}{1 - 2x}$ for $x \neq \frac{1}{2}, 1$.

(b) Obtain f^3 in a similar form.

7. A function h is defined by $h : x \mapsto \frac{x + 3}{x - 3}$ (for $x \neq 3$).

(a) Show that $h(3 + p) + h(3 - p) = 2$ where p is a positive number.

(b) Find the positive number q such that $h(q) = q - 1$.

8. Functions f and g are defined by

$$f: x \mapsto 4x - 3,$$
$$g: x \mapsto px + q, \text{ where } p > 0.$$

Find the values of p and q for which $gg(x) = f(x)$ for all values of x.

9. Functions f and g are defined by

$$f: x \mapsto ax + b, \text{ where } a \text{ and } b \text{ are constants,}$$
$$g: x \mapsto \frac{3}{x-2}, \quad x \neq 2.$$

Find expressions for fg and gf. Given that $fg(5) = 5$ and $g^{-1}(2) = f(2)$, calculate the value of a and of b.

10. The functions f and g are defined by

$$f: x \mapsto \frac{3x+2}{x}, \quad x \neq 0,$$
$$g: x \mapsto \frac{1}{x}, \quad x \neq 0.$$

Show that

(a) $f(x) = 2g(x) + 3$, **(b)** $f^2(x) = 2gf(x) + 3$.

11. Sketch the graph and find the range of each of the following functions for the domain $-2 \leqslant x \leqslant 3$.

(a) $f: x \mapsto |2x + 1|$ **(b)** $g: x \mapsto x^2 - x - 2$

(c) $h: x \mapsto |x^2 - x|$ **(d)** $k: x \mapsto 3 - |x^2 - 2|$

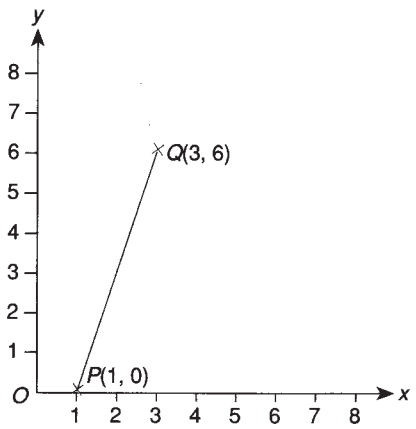
12. For each of the following functions, state the range that corresponds to the given domain.

(a) $f(x) = |2x - 9|$ for the domain $0 \leqslant x \leqslant 6$

(b) $g(x) = (x - 2)^2 - 3$ for the domain $0 \leqslant x \leqslant 5$

(c) $h(x) = |2 - 3(x - 1)^2|$ for the domain $-1 < x < 2$

13. The diagram shows the graph of $y = f(x)$. The scale is the same on each axis. Copy the diagram and on it sketch the graph of $y = f^{-1}(x)$, indicating the point on $y = f^{-1}(x)$ corresponding to the points P and Q. State the relationship between the graphs of $y = f(x)$ and $y = f^{-1}(x)$. Hence calculate the element whose images under f and f^{-1} are the same.



14. Functions f, g and h are defined by

$$f: x \mapsto 1 - 2x,$$
$$g: x \mapsto |x + 1|,$$
$$h: x \mapsto x^2 - 2.$$

(a) Evaluate $gh(-4)$ and $f^{-1}gh(-4)$.
(b) Find in similar form, gf, fg and gh.
(c) Find the elements whose images under fg are -3.
(d) Show that $hg: x \mapsto x^2 + 2x - 1$.

15. Functions f and g are defined by $f: x \mapsto \dfrac{x+2}{x-2}$, $x \neq 2$, and $g: x \mapsto mx + c$, where m and c are constants.
(a) Obtain an expression for f^{-1}.
(b) Given that $g^{-1}(2) = f(3)$ and $gf^{-1}(2) = 5$, find the value of m and of c.

16. A function f is defined by $f: x \mapsto |x^2 - a| + b$ where a and b are constants. Given that $f(-2) = b$ and $f(3) = 8$, find the value of a and of b. Sketch the graph of f for $-2 \leqslant x \leqslant 3$ and state the corresponding range.

17. Functions f and g are defined by

$$f: x \mapsto e^{x-2},$$
$$g: x \mapsto \frac{1}{3} \ln x, \; x > 0.$$

(a) Show that $f^{-1}: x \mapsto \ln x + 2$, (for $x > 0$).
(b) Find in similar form, g^{-1} and gf.

Miscellaneous Exercise 8 *(p. 151)*

1. **(a)** $p = 3,\ q = -1$ **(b)** 8 **(c)** 3 **2.** $a = 3;\ x = -\frac{1}{3}$

3. **(a)** $f^{-1}: x \mapsto 2 - x$ **(b)** $ff: x \mapsto x$ **(c)** $gg: x \mapsto x,\ x \neq 0$

(d) $fg: x \mapsto \dfrac{2x-3}{x},\ x \neq 0$ **(e)** $gfg: x \mapsto \dfrac{3x}{2x-3},\ x \neq 0, \frac{3}{2}$

4. **(a)** $ff: x \mapsto 16x - 15$ **(b)** $gf: x \mapsto \dfrac{8x-11}{4x-3},\ x \neq \frac{3}{4}$

(c) $f^{-1}: x \mapsto \frac{1}{4}(x+3)$ **(d)** $g^{-1}: x \mapsto \dfrac{5}{2-x},\ x \neq 2$

(e) $(fg)^{-1}: x \mapsto \dfrac{20}{5-x},\ x \neq 5$

5. **(a)** 7 **(b)** -5 **(c)** 2 **(d)** $g^{-1}: x \mapsto \dfrac{3-x}{x+1},\ x \neq -1$

(e) $fg: x \mapsto -\dfrac{2x}{x+1},\ x \neq -1$ **(f)** $gfg: x \mapsto \dfrac{5x+3}{1-x},\ x \neq -1,\ 1$

6. $f^3: x \mapsto \dfrac{x}{1-3x},\ x \neq 1, \frac{1}{2}, \frac{1}{3}$ **7.** **(b)** $q = 5$ **8.** $p = 2,\ q = -1.$

9. $fg: x \mapsto \dfrac{3a}{x-2} + b,\ x \neq 2;\ gf: x \mapsto \dfrac{3}{ax+b-2},\ x \neq \dfrac{2-b}{a};\ a = -\frac{3}{2},\ b = \frac{13}{2}$

11. **(a)** $0 \leqslant f(x) \leqslant 7$ **(b)** $-2\frac{1}{4} \leqslant g(x) \leqslant 4$ **(c)** $0 \leqslant h(x) \leqslant 6$ **(d)** $-4 \leqslant k(x) \leqslant 3$

12. **(a)** $0 \leqslant f(x) \leqslant 9$ **(b)** $-3 \leqslant g(x) \leqslant 6$ **(c)** $0 \leqslant h(x) < 10$

13. The graphs are reflective images of each other in the line $y = x$; $1\frac{1}{2}$

14. **(a)** 15, -7 **(b)** $gf: x \mapsto |2 - 2x|$; $fg: x \mapsto 1 - 2|x+1|$; $gh: x \mapsto |x^2 - 1|$
(c) $-3,\ 1$

15. **(a)** $f^{-1}: x \mapsto \dfrac{2(x+1)}{x-1},\ x \neq 1$ **(b)** $m = 3,\ c = -13$

16. $a = 4,\ b = 3,\ 3 \leqslant y \leqslant 8$

17. **(b)** $g^{-1}: x \mapsto e^{3x}$; $gf: x \mapsto \frac{1}{3}(x-2)$