15-09-14-S12.pdf

EXERCISES 1.3

1–12 ■ Evaluate the given limit and justify each step by indicating the appropriate Limit Law(s).

1. $\lim_{x\to 4}(5x^2 - 2x + 3)$
2. $\lim_{x\to -3}(x^3 + 2x^2 + 6)$
3. $\lim_{x\to 2}(x^2 + 1)(x^2 + 4x)$
4. $\lim_{x\to -2}(x^2 + x + 1)^5$
5. $\lim_{x\to -1}\dfrac{x - 2}{x^2 + 4x - 3}$
6. $\lim_{t\to -2}\dfrac{t^3 - t^2 - t + 10}{t^2 + 3t + 2}$
7. $\lim_{x\to -1}\sqrt{x^3 + 2x + 7}$
8. $\lim_{x\to 64}(\sqrt[3]{x} + 3\sqrt{x})$
9. $\lim_{t\to -2}(t + 1)^9(t^2 - 1)$
10. $\lim_{r\to 3}(r^4 - 7r + 4)^{2/3}$
11. $\lim_{w\to -2}\sqrt[3]{\dfrac{4w + 3w^3}{3w + 10}}$
12. $\lim_{y\to 3}\dfrac{3(8y^2 - 1)}{2y^2(y - 1)^4}$
13. Given that

$$\lim_{x\to a} f(x) = -3 \qquad \lim_{x\to a} g(x) = 0 \qquad \lim_{x\to a} h(x) = 8$$

find the limits that exist.

(a) $\lim_{x\to a}[f(x) + h(x)]$ (b) $\lim_{x\to a}[f(x)]^2$

(c) $\lim_{x\to a}\sqrt[3]{h(x)}$ (d) $\lim_{x\to a}\dfrac{1}{f(x)}$

(e) $\lim_{x\to a}\dfrac{f(x)}{h(x)}$ (f) $\lim_{x\to a}\dfrac{g(x)}{f(x)}$

(g) $\lim_{x\to a}\dfrac{f(x)}{g(x)}$ (h) $\lim_{x\to a}\dfrac{2f(x)}{h(x) - f(x)}$

14. (a) What is wrong with the equation $\dfrac{x^2 + x - 6}{x - 2} = x + 3$?

(b) In view of part (a), explain why the equation

$$\lim_{x\to 2}\frac{x^2 + x - 6}{x - 2} = \lim_{x\to 2}(x + 3)$$

is correct.

15–38 ■ Evaluate each limit, if it exists.

15. $\lim_{x\to -3}\dfrac{x^2 - x + 12}{x + 3}$
16. $\lim_{x\to -3}\dfrac{x^2 - x - 12}{x + 3}$
17. $\lim_{x\to -1}\dfrac{x^2 - x - 2}{x + 1}$
18. $\lim_{x\to 1}\dfrac{x^2 - x - 2}{x + 1}$
19. $\lim_{t\to 1}\dfrac{t^3 - t}{t^2 - 1}$
20. $\lim_{x\to -1}\dfrac{x^2 - x - 3}{x + 1}$
21. $\lim_{h\to 0}\dfrac{(h - 5)^2 - 25}{h}$
22. $\lim_{x\to 1}\dfrac{x^3 - 1}{x^2 - 1}$
23. $\lim_{h\to 0}\dfrac{(1 + h)^4 - 1}{h}$
24. $\lim_{h\to 0}\dfrac{(2 + h)^3 - 8}{h}$
25. $\lim_{x\to -2}\dfrac{x + 2}{x^2 - x - 6}$
26. $\lim_{x\to 1}\dfrac{x^2 + x - 2}{x^2 - 3x + 2}$
27. $\lim_{t\to 9}\dfrac{9 - t}{3 - \sqrt{t}}$
28. $\lim_{t\to 2}\dfrac{t^2 + t - 6}{t^2 - 4}$
29. $\lim_{t\to 0}\dfrac{\sqrt{2 - t} - \sqrt{2}}{t}$
30. $\lim_{x\to 2}\dfrac{x^4 - 16}{x - 2}$
31. $\lim_{x\to 9}\dfrac{x^2 - 81}{\sqrt{x} - 3}$
32. $\lim_{x\to 1}\left[\dfrac{1}{x - 1} - \dfrac{2}{x^2 - 1}\right]$
33. $\lim_{t\to 0}\left[\dfrac{1}{t\sqrt{1 + t}} - \dfrac{1}{t}\right]$
34. $\lim_{h\to 0}\dfrac{(3 + h)^{-1} - 3^{-1}}{h}$
35. $\lim_{x\to 0}\dfrac{x}{\sqrt{1 + 3x} - 1}$
36. $\lim_{x\to 2}\dfrac{\dfrac{1}{x} - \dfrac{1}{2}}{x - 2}$
37. $\lim_{x\to 2}\dfrac{x - \sqrt{3x - 2}}{x^2 - 4}$
38. $\lim_{x\to 1}\dfrac{\sqrt{x} - x^2}{1 - \sqrt{x}}$

39. Use the Squeeze Theorem to show that $\lim_{x\to 0} x^2\cos 20\pi x = 0$. Illustrate by graphing the functions $f(x) = -x^2$, $g(x) = x^2\cos 20\pi x$, and $h(x) = x^2$ on the same screen.

40. Use the Squeeze Theorem to show that $\lim_{x\to 0}\sqrt{x^3 + x^2}\,\sin(\pi/x) = 0$. Illustrate by graphing the functions f, g, and h (in the notation of the Squeeze Theorem) on the same screen.

41. If $1 \le f(x) \le x^2 + 2x + 2$ for all x, find $\lim_{x\to -1} f(x)$.

42. If $3x \le f(x) \le x^3 + 2$ for $0 \le x \le 2$, evaluate $\lim_{x\to 1} f(x)$.

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1. 75 **3.** 60 **5.** $\frac{1}{2}$ **7.** 2 **9.** -3 **11.** -2

13. (a) 5 (b) 9 (c) 2 (d) $-\frac{1}{3}$ (e) $-\frac{3}{8}$ (f) 0
(g) Does not exist (h) $-\frac{6}{11}$

15. Does not exist **17.** -3 **19.** 1 **21.** -10 **23.** 4

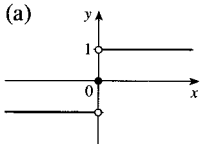
25. $-\frac{1}{5}$ **27.** 6 **29.** $-\sqrt{2}/4$ **31.** 108 **33.** $-\frac{1}{2}$

35. $\frac{2}{3}$ **37.** $\frac{1}{16}$ **41.** 1 **45.** 0 **47.** 0

49. Does not exist **51.** -2 **53.** -3 **55.** 0

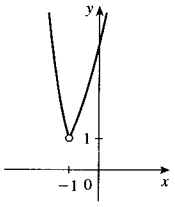
57. Does not exist

59. (a)



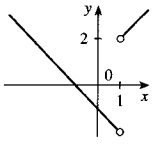
(b) (i) 1 (ii) -1
(iii) Does not exist (iv) 1

61. (a) 1, 1
(b) Yes, 1
(c)



63. (a) (i) $n - 1$ (ii) n
(b) a is not an integer

65. (a) (i) 2 (ii) -2
(b) No
(c)



73. $c/3$ **75.** 15, -1