15-11-14-Th

EXERCISES 3.8

In Exercises 1–4, find the linearization $L(x)$ of $f(x)$ at $x = a$.

1. $f(x) = x^3 - 2x + 3, \quad a = 2$
2. $f(x) = \sqrt{x^2 + 9}, \quad a = -4$
3. $f(x) = x + \frac{1}{x}, \quad a = 1$
4. $f(x) = \sqrt[3]{x}, \quad a = -8$

You want linearizations that will replace the functions in Exercises 5–10 over intervals that include the given points x_0. To make your subsequent work as simple as possible, you want to center each linearization not at x_0 but at a nearby integer $x = a$ at which the given function and its derivative are easy to evaluate. What linearization do you use in each case?

5. $f(x) = x^2 + 2x, \quad x_0 = 0.1$
6. $f(x) = x^{-1}, \quad x_0 = 0.9$
7. $f(x) = 2x^2 + 4x - 3, \quad x_0 = -0.9$
8. $f(x) = 1 + x, \quad x_0 = 8.1$
9. $f(x) = \sqrt[3]{x}, \quad x_0 = 8.5$
10. $f(x) = \frac{x}{x + 1}, \quad x_0 = 1.3$

More problems on next page

Answers below.

Section 3.8, pp. 231–234

1. $L(x) = 10x - 13$ **3.** $L(x) = 2$ **5.** $2x$ **7.** -5

9. $\frac{1}{12}x + \frac{4}{3}$ **11. (a)** $L(x) = x$ **(b)** $L(x) = \pi - x$

13. (a) $L(x) = 1$ **(b)** $L(x) = 2 - 2\sqrt{3}\left(x + \frac{\pi}{3}\right)$

15. $f(0) = 1$. Also, $f'(x) = k(1 + x)^{k-1}$, so $f'(0) = k$. This means the linearization at $x = 0$ is $L(x) = 1 + kx$.

17. (a) 1.01 **(b)** 1.003

19. $\left(3x^2 - \frac{3}{2\sqrt{x}}\right) dx$ **21.** $\frac{2 - 2x^2}{(1 + x^2)^2} dx$

23. $\frac{1 - y}{3\sqrt{y} + x} dx$ **25.** $\frac{5}{2\sqrt{x}} \cos(5\sqrt{x})\, dx$

27. $(4x^2) \sec^2\left(\frac{x^3}{3}\right) dx$

29. $\frac{3}{\sqrt{x}}(\csc(1 - 2\sqrt{x}) \cot(1 - 2\sqrt{x}))\, dx$

31. (a) .41 **(b)** .4 **(c)** .01

33. (a) .231 **(b)** .2 **(c)** .031

35. (a) $-1/3$ **(b)** $-2/5$ **(c)** $1/15$

37. $dV = 4\pi r_0^2\, dr$ **39.** $dS = 12x_0\, dx$ **41.** $dV = 2\pi r_0 h\, dr$

43. (a) 0.08π m^2 **(b)** 2% **45.** $dV \approx 565.5$ in.^{3}

47. $\frac{1}{3}\%$ **49.** 0.05%

51. The ratio equals 37.87, so a change in the acceleration of gravity on the moon has about 38 times the effect that a change of the same magnitude has on Earth.

53. 3% **55.** 3%

59. $\lim_{x \to 0} \frac{\sqrt{1 + x}}{1 + \left(\frac{x}{2}\right)} = \frac{\sqrt{1 + 0}}{1 + \left(\frac{0}{2}\right)} = \frac{1}{1} = 1$ **65.** $0.07c$

Linearizing Trigonometric Functions

In Exercises 11–14, find the linearization of f at $x = a$. Then graph the linearization and f together.

11. $f(x) = \sin x$ at (a) $x = 0$, (b) $x = \pi$

12. $f(x) = \cos x$ at (a) $x = 0$, (b) $x = -\pi/2$

13. $f(x) = \sec x$ at (a) $x = 0$, (b) $x = -\pi/3$

14. $f(x) = \tan x$ at (a) $x = 0$, (b) $x = \pi/4$

The Approximation $(1 + x)^k \approx 1 + kx$

15. Show that the linearization of $f(x) = (1 + x)^k$ at $x = 0$ is $L(x) = 1 + kx$.

16. Use the linear approximation $(1 + x)^k \approx 1 + kx$ to find an approximation for the function $f(x)$ for values of x near zero.

a. $f(x) = (1 - x)^6$ **b.** $f(x) = \dfrac{2}{1 - x}$

c. $f(x) = \dfrac{1}{\sqrt{1 + x}}$ **d.** $f(x) = \sqrt{2 + x^2}$

e. $f(x) = (4 + 3x)^{1/3}$ **f.** $f(x) = \sqrt[3]{\left(1 - \dfrac{1}{2 + x}\right)^2}$

17. Faster than a calculator Use the approximation $(1 + x)^k \approx 1 + kx$ to estimate the following.

a. $(1.0002)^{50}$ **b.** $\sqrt[3]{1.009}$

18. Find the linearization of $f(x) = \sqrt{x + 1} + \sin x$ at $x = 0$. How is it related to the individual linearizations of $\sqrt{x + 1}$ and $\sin x$ at $x = 0$?

Derivatives in Differential Form

In Exercises 19–30, find dy.

19. $y = x^3 - 3\sqrt{x}$ **20.** $y = x\sqrt{1 - x^2}$

21. $y = \dfrac{2x}{1 + x^2}$ **22.** $y = \dfrac{2\sqrt{x}}{3(1 + \sqrt{x})}$

23. $2y^{3/2} + xy - x = 0$ **24.** $xy^2 - 4x^{3/2} - y = 0$

25. $y = \sin(5\sqrt{x})$ **26.** $y = \cos(x^2)$

27. $y = 4\tan(x^3/3)$ **28.** $y = \sec(x^2 - 1)$

29. $y = 3\csc(1 - 2\sqrt{x})$ **30.** $y = 2\cot\left(\dfrac{1}{\sqrt{x}}\right)$

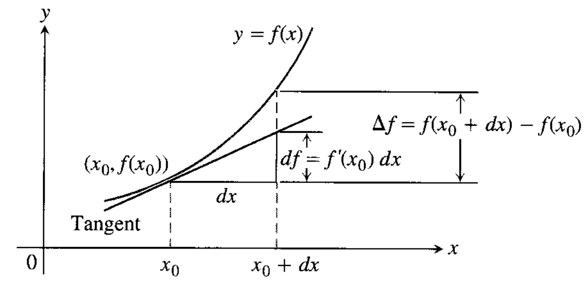
Appr[illegible]

In Exercises 31–36, each function $f(x)$ changes value when x changes from x_0 to $x_0 + dx$. Find

a. the change $\Delta f = f(x_0 + dx) - f(x_0)$;

b. the value of the estimate $df = f'(x_0)\,dx$; and

c. the approximation error $|\Delta f - df|$.



31. $f(x) = x^2 + 2x$, $x_0 = 1$, $dx = 0.1$

32. $f(x) = 2x^2 + 4x - 3$, $x_0 = -1$, $dx = 0.1$

33. $f(x) = x^3 - x$, $x_0 = 1$, $dx = 0.1$

34. $f(x) = x^4$, $x_0 = 1$, $dx = 0.1$

35. $f(x) = x^{-1}$, $x_0 = 0.5$, $dx = 0.1$

36. $f(x) = x^3 - 2x + 3$, $x_0 = 2$, $dx = 0.1$

Differential Estimates of Change

In Exercises 37–42, write a differential formula that estimates the given change in volume or surface area.

37. The change in the volume $V = (4/3)\pi r^3$ of a sphere when the radius changes from r_0 to $r_0 + dr$

38. The change in the volume $V = x^3$ of a cube when the edge lengths change from x_0 to $x_0 + dx$

39. The change in the surface area $S = 6x^2$ of a cube when the edge lengths change from x_0 to $x_0 + dx$

40. The change in the lateral surface area $S = \pi r\sqrt{r^2 + h^2}$ of a right circular cone when the radius changes from r_0 to $r_0 + dr$ and the height does not change

41. The change in the volume $V = \pi r^2 h$ of a right circular cylinder when the radius changes from r_0 to $r_0 + dr$ and the height does not change

42. The change in the lateral surface area $S = 2\pi rh$ of a right circular cylinder when the height changes from h_0 to $h_0 + dh$ and the radius does not change

Applications

43. The radius of a circle is increased from 2.00 to 2.02 m.

a. Estimate the resulting change in area.

b. Express the estimate as a percentage of the circle's original area.

44. The diameter of a tree was 10 in. During the following year, the circumference increased 2 in. About how much did the tree's diameter increase? The tree's cross-section area?

45. Estimating volume Estimate the volume of material in a cylindrical shell with height 30 in., radius 6 in., and shell thickness 0.5 in.