Derivative of absolute value function

If $h(x) = |f(x)|$ is differentiable and $f(x) \neq 0$ on some interval I, then for all $x \in I$,

$$h'(x) = \frac{|f(x)|}{f(x)} f'(x). \tag{1.1}$$

Keep in mind that $|f(x)|$ is a composed function, so the chain rule applies. The "outer" function is the absolute value function and the "inner" function is $f(x)$. We could write "abs$(f(x))$" instead of "$|f(x)|$".

To see why equation 1.1 is true, consider two cases because

$$|f(x)| = \begin{cases} f(x), & \text{if } f(x) > 0 \\ -f(x), & \text{if } f(x) < 0. \end{cases}$$

Case 1. $f(x) > 0$. Then $|f(x)| = f(x)$. So that

$$h'(x) = f'(x) = \frac{f(x)}{f(x)} f'(x) = \frac{|f(x)|}{f(x)} f'(x).$$

Case 2. $f(x) < 0$. Then $|f(x)| = -f(x)$. Thus

$$h'(x) = -f'(x) = \frac{-f(x)}{f(x)} f'(x) = \frac{|f(x)|}{f(x)} f'(x).$$

In either case, we have

$$h'(x) = \frac{|f(x)|}{f(x)} f'(x).$$