15-10-6-S13

EXERCISES 10.9

1–8 ■ Find a power series representation for the function and determine the interval of convergence.

1. $f(x) = \dfrac{1}{1+x}$
2. $f(x) = \dfrac{x}{1-x}$
3. $f(x) = \dfrac{1}{1+4x^2}$
4. $f(x) = \dfrac{1}{x^4+16}$
5. $f(x) = \dfrac{1}{4+x^2}$
6. $f(x) = \dfrac{1+x^2}{1-x^2}$
7. $f(x) = \dfrac{x}{x-3}$
8. $f(x) = \dfrac{2}{3x+4}$

9–10 ■ Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

9. $f(x) = \dfrac{3x-2}{2x^2-3x+1}$
10. $f(x) = \dfrac{x}{x^2-3x+2}$

11–18 ■ Find a power series representation for the function and determine the radius of convergence.

11. $f(x) = \dfrac{1}{(1+x)^2}$
12. $f(x) = \ln(1+x)$
13. $f(x) = \dfrac{1}{(1+x)^3}$
14. $f(x) = x\ln(1+x)$
15. $f(x) = \ln(5-x)$
16. $f(x) = \tan^{-1}(2x)$
17. $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$
18. $f(x) = \dfrac{x^2}{(1-2x)^2}$

19–20 ■ Find a power series representation for f and graph f and several partial sums $s_n(x)$ on the same screen. What happens as n increases?

19. $f(x) = \ln(3+x)$
20. $f(x) = \dfrac{1}{x^2+25}$

21–24 ■ Evaluate the indefinite integral as a power series.

21. $\displaystyle\int \frac{1}{1+x^4}\,dx$
22. $\displaystyle\int \frac{x}{1+x^5}\,dx$
23. $\displaystyle\int \frac{\arctan x}{x}\,dx$
24. $\displaystyle\int \tan^{-1}(x^2)\,dx$

25–28 ■ Use a power series to approximate the definite integral to six decimal places.

25. $\displaystyle\int_0^{0.2} \frac{1}{1+x^4}\,dx$
26. $\displaystyle\int_0^{1/2} \tan^{-1}(x^2)\,dx$
27. $\displaystyle\int_0^{1/3} x^2\tan^{-1}(x^4)\,dx$
28. $\displaystyle\int_0^{0.5} \frac{dx}{1+x^6}$

29. Use the result of Example 6 to compute ln 1.1 correct to five decimal places.

30. Show that the function

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

is a solution of the differential equation $f''(x) + f(x) = 0$.

31. (a) Show that J_0 (the Bessel function of order 0 given in Example 4) satisfies the differential equation

$$x^2 J_0''(x) + xJ_0'(x) + x^2 J_0(x) = 0$$

(b) Evaluate $\int_0^1 J_0(x)\,dx$ correct to three decimal places.

32. The Bessel function of order 1 is defined by

$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$$

(a) Show that J_1 satisfies the differential equation

$$x^2 J_1''(x) + xJ_1'(x) + (x^2-1)J_1(x) = 0$$

(b) Show that $J_0'(x) = -J_1(x)$.

33. (a) Show that the function $f(x) = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}$ is a solution of the differential equation $f'(x) = f(x)$.
(b) Show that $f(x) = e^x$.

34. Let $f_n(x) = (\sin nx)/n^2$. Show that the series $\Sigma f_n(x)$ converges for all values of x but the series of derivatives $\Sigma f_n'(x)$ diverges when $x = 2n\pi$, n an integer. For what values of x does the series $\Sigma f_n''(x)$ converge?

35. Let

$$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$$

Find the intervals of convergence for f, f', and f''.

36. (a) Starting with the geometric series $\Sigma_{n=0}^{\infty} x^n$, find the sum of the series

$$\sum_{n=1}^{\infty} nx^{n-1} \qquad |x| < 1$$

(b) Find the sums of the following series.
(i) $\displaystyle\sum_{n=1}^{\infty} nx^n$, $|x| < 1$ (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

(c) Find the sums of the following series.
(i) $\displaystyle\sum_{n=2}^{\infty} n(n-1)x^n$, $|x| < 1$ (ii) $\displaystyle\sum_{n=2}^{\infty} \frac{n^2-n}{2^n}$
(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

Exercises 10.5 ■ page 613

1. C 3. D 5. C 7. D 9. C 11. C 13. D 15. C 17. C 19. D 21. $\{b_n\}$ is not decreasing 23. p is not a negative integer 25. 0.82 27. 0.13 (or 0.137) 29. 0.8415 31. 0.6065 33. An underestimate

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Abbreviations: *AC, absolutely convergent*; *CC, conditionally convergent*

1. AC 3. D 5. CC 7. AC 9. CC 11. D 13. AC 15. AC 17. AC 19. D 21. AC 23. D 25. AC 27. AC 29. D 31. AC 33. D 35. (a) and (d) 39. (a) $\frac{661}{960} \approx 0.68854$, error < 0.00521 (b) $n \geqslant 11$, 0.693109

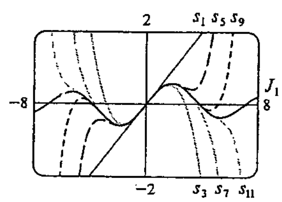
Exercises 10.7 ■ page 622

1. C 3. C 5. C 7. C 9. C 11. D 13. C 15. C 17. C 19. C 21. D 23. D 25. C 27. D 29. C 31. C 33. C 35. C 37. C 39. C

Exercises 10.8 ■ page 627

1. (a) Yes (b) No 3. 1, $[-1, 1)$ 5. 1, $(-1, 1)$ 7. ∞, $(-\infty, \infty)$ 9. 2, $(-2, 2]$ 11. $\frac{1}{3}$, $[-\frac{1}{3}, \frac{1}{3}]$ 13. 1, $[-1, 1)$ 15. 2, $(-\frac{3}{2}, \frac{5}{2})$ 17. 1, $(0, 2]$ 19. ∞, $(-\infty, \infty)$ 21. 0.5, $[2.5, 3.5)$ 23. 0, $\{-6\}$ 25. $\frac{1}{2}$, $[0, 1]$ 27. ∞, $(-\infty, \infty)$ 29. k^k

31. (a) $(-\infty, \infty)$
(b), (c)



33. $(-1, 1)$, $f(x) = (1 + 2x)/(1 - x^2)$ 37. 2

Exercises 10.9 ■ page 632

1. $\sum_{n=0}^{\infty} (-1)^n x^n$, $(-1, 1)$ 3. $\sum_{n=0}^{\infty} (-1)^n 4^n x^{2n}$, $(-\frac{1}{2}, \frac{1}{2})$

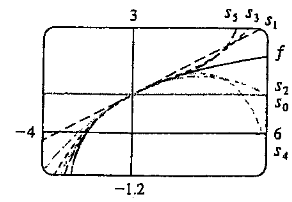
5. $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{n+1}} x^{2n}$, $(-2, 2)$ 7. $-\sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n$, $(-3, 3)$

9. $-\sum_{n=0}^{\infty} (2^n + 1)x^n$, $(-\frac{1}{2}, \frac{1}{2})$ 11. $\sum_{n=0}^{\infty} (-1)^n (n + 1)x^n$, $R = 1$

13. $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n (n + 2)(n + 1)x^n$, $R = 1$

15. $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}$, $R = 5$ 17. $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n + 1}$, $R = 1$

19. $\ln 3 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n3^n} x^n$, $R = 3$
The partial sums approximate f better (on the interval of convergence).



21. $C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{4n + 1}$ 23. $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n + 1)^2}$

25. 0.199936 27. 0.000065 29. 0.09531 31. (b) 0.920 35. $[-1, 1]$, $[-1, 1)$, $(-1, 1)$

Exercises 10.10 ■ page 643

1. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$, $R = \infty$ 3. $\sum_{n=0}^{\infty} (-1)^n (n + 1)x^n$, $R = 1$

5. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n + 1)!}$, $R = \infty$

7. $\sum_{n=0}^{\infty} \frac{(-1)^{n(n-1)/2}(x - \pi/4)^n}{\sqrt{2}\,n!}$, $R = \infty$

9. $\sum_{n=0}^{\infty} (-1)^n (x - 1)^n$, $R = 1$ 11. $\sum_{n=0}^{\infty} \frac{e^3}{n!}(x - 3)^n$, $R = \infty$

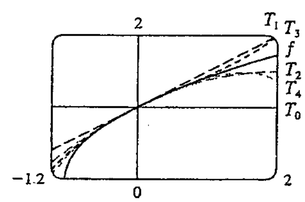
17. $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$, $R = \infty$ 19. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n)!}$, $R = \infty$

21. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{2^{2n+1}(2n + 1)!}$, $R = \infty$

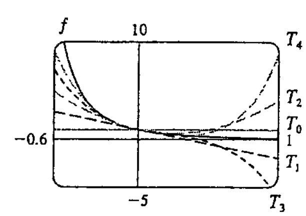
23. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}$, $R = \infty$

25. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n + 1)!}$, $R = \infty$

27. $1 + \frac{x}{2} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 3)}{2^n n!} x^n$, $R = 1$



29. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2} (n + 1)(n + 2)x^n$,
$R = 1$



31. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$, 0.09531 33. $C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+3}}{(4n + 3)(2n + 1)!}$

35. $C + x + \frac{x^4}{8} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n - 3)}{2^n n! (3n + 1)} x^{3n+1}$