Not such a great moment in the history of arithmetic

I wish I had not bobbled comparing $\frac{9\sqrt{3}}{2} + \frac{9\sqrt{3}}{4}$ with 9 today, since the point of the "demonstration" was to make it look easier than reaching for a calculator. Allow me to make amends.

$$\frac{9\sqrt{3}}{2} + \frac{9\sqrt{3}}{4} = 9\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right)$$

$$= 9\left(\frac{3\sqrt{3}}{4}\right).$$

So, the question becomes Is $\frac{3\sqrt{3}}{4} \lesseqqgtr 1$? Since

$$\left(\frac{3\sqrt{3}}{4}\right)^2 = \frac{27}{16} > 1,$$

we conclude $\frac{3\sqrt{3}}{4} > 1$, too.

This argument relies on the fact that for positive numbers a and b,

$$a > b \iff a^2 > b^2. \tag{1}$$

You should be able to prove statement (1) by elementary means. You should also be able to prove statement (1) using the so-called "monotonicity theorem" which says that if f is continuous and differentiable at every interior point of I, then $f'(x) > 0 \implies f$ is increasing on I and $f'(x) < 0 \implies f$ is decreasing on I.

Exercise

1. Prove equation (1) by elementary means.
2. Use the monotonicity theorem to prove equation (1).