1. For each of the following GPs, find the common ratio, r.
   (a) 3, 9, 27, . . .
   (b) 10, 5, $2\frac{1}{2}$, . . .
   (c) $\sqrt{3}$, $3\sqrt{2}$, $6\sqrt{3}$, . . .
   (d) x, $x^2 + x$, $x(x + 1)^2$, . . .

2. Find the indicated terms of these GPs.
   (a) 2, 10, 50, . . . ; T_5 and T_7
   (b) $\frac{1}{8}$, $\frac{1}{2}$, 2, . . . ; T_6 and T_{10}

3. Find the fifth term of the GP $\frac{1}{27}$, $\frac{1}{9}$, $\frac{1}{3}$, . . . , 81. How many terms are there?

4. In a GP the fourth term is 18 and the seventh term is 486. Find the second term.

5. The terms of a GP are distinct. If the second term is 4 and the sum of its third and fourth terms is 8, find the sixth term.

6. In a GP whose terms are all positive, the third term exceeds the first term by 32 while the fifth term exceeds the first term by 320. Calculate the first term.

7. Find the geometric means of the following:
   (a) 3, 12
   (b) $\frac{1}{3}$, $\frac{4}{27}$
   (c) $\sqrt{24}$, $\sqrt{6}$

8. Find the indicated sums of these GPs.
   (a) 5, 10, 20, . . . ; S_5 and S_7
   (b) 2, −4, 8, . . . ; S_6 and S_9

9. Given x, $x + 4$, $2x + 2$ are the first three positive terms of a GP, calculate the sum of the first 6 terms.

10. The sum of the first 4 terms of a GP is 40. Its third term exceeds the first term by 8. If the first term is greater than 1, calculate the sum of the first 6 terms.

11. The sum of the first 3 terms of a GP is 117 and the sum of the next 3 terms is $4\frac{1}{3}$. Calculate the third term.

12. Find the least number of terms of the GP 2, 4, 8, . . . which must be taken for their sum to exceed 2 500.

13. The second term of a GP is 1 less than the first term and the sum of the first 3 terms is $6\frac{1}{3}$. If the GP contains positive and negative terms, find the seventh term.

14. A GP has first term 5 and last term 2 560. If the sum of all its terms is 5 115, how many terms are there?

15. The r^{th} term of a GP is $\frac{6 \times 5^{r-1}}{2^r}$. Find the common ratio and an expression for the sum of the first $2n$ terms.

16. Find three numbers in a GP such that their sum is 42 and their product is 512.

17. The sum of the first $(n + 10)$ terms of the geometric series $4 + 8 + 16 + \ldots$ is twice the sum of the first n terms of the geometric series $6 + 24 + 96 + \ldots$. Calculate the value of n.

18. In the GP 3, 6, 12, . . . , the r^{th} term is the largest term in the progression which has a value less than 1 000. Find r.

19. The sum of the first 4 terms of a GP is 5 times the sum of its first 2 terms. If the fifth term is 128, find the possible values of the eighth term.

20. The sum of the first n terms of a GP is $k(3^{n-1}) - 9$. If the third term is 162, find k and the common ratio.

21. The lengths of the sides of a quadrilateral are in geometric progression and the longest side is 81 cm. Given that the perimeter is 120 cm, find the lengths of the other 3 sides.

***22.** A GP has 10 terms. The sum of the whole series is 5 9 04.8 while the sum of all the even-numbered terms (2^{nd}, 4^{th}, 6^{th}, etc.) is 4 428.6. Find the sum of the 4 middle terms.

***23.** If k is a positive integer and $x \neq -1, 1$, prove that
$(1 + x + x^2 + x^3 + \ldots + x^{2k})(1 - x + x^2 - x^3 + \ldots + x^{2k}) = 1 + x^2 + x^4 + \ldots + x^{4k}$.

Exercise 13.3 *(p. 259)*

1. (a) 3 (b) $\frac{1}{2}$ (c) $\sqrt{6}$ (d) $x + 1$

2. (a) 1 250, 31 250 (b) 128, 32 768 **3.** 3; 8 **4.** 2

5. 64 **6.** 4 **7.** (a) 6 (b) $\frac{2}{9}$ (c) $2\sqrt{3}$

8. (a) 155, 635 (b) -42, 342 **9.** $166\frac{1}{4}$ **10.** 168

11. 9 **12.** 11 **13.** $29\,127\frac{1}{9}$ **14.** 10

15. $\frac{5}{2}$, $2\left[\left(\frac{5}{2}\right)^{2n} - 1\right]$ **16.** 32, 8, 2 **17.** 10 **18.** 9

19. $\pm 1\,024$ **20.** 27, 3 **21.** 3 cm, 9 cm, 27 cm **22.** 216