

Solutions that are typeset have been check

Others may or may not have been checke

P68

$$\checkmark [1.1] \quad a_{20} = 4 + 19(3) = 61$$

$$\checkmark [1.2] \quad a_{25} = 7 + 24\left(-\frac{1}{3}\right) = -1$$

$$\checkmark [2.1] \quad a = 23, d = (36 - 23) = 7 \Rightarrow a_n = 23 + (n-1)7$$

$$\checkmark [2.2] \quad a = 2, d = \frac{5}{4} - 2 = -\frac{3}{4} \Rightarrow a_n = 2 + (n-1)\left(-\frac{3}{4}\right)$$

P69

$$\checkmark [3] \quad -37 = 8 + (n-1)(-3) \Rightarrow n = 16 \quad \therefore a_{16}$$

$$\checkmark [4] \quad \begin{matrix} a_3 = -4 \\ a_{10} = 38 \end{matrix} \Rightarrow \begin{bmatrix} a + 9d = 38 \\ a + 2d = -4 \end{bmatrix} \Rightarrow a_n = -16 + (n-1)6$$

$$\checkmark [5] \quad a = 3, a_{15} = 94,$$

$$3 + 14d = 94 \Rightarrow d = \frac{13}{2}$$

$$a_5 = 3 + 4\left(\frac{13}{2}\right) = 29$$

$$a_{16} = 3 + 9\left(\frac{13}{2}\right) = \frac{123}{2}$$

P70

$\checkmark [6]$  Difference of sequential terms is  $a_{n+1} - a_n$ ,

$$[p(n+1) + q] - [pn + q] = p$$

Since by hypothesis,  $p$  constant,  $\{pn + q\}$  is arithmetic sequence.

$$\checkmark [7] \quad a_n = a + (n-1)d_a \text{ and } b_n = b + (n-1)d_b$$

$$a_n + b_n = a + da_n - da + b + db_n - db$$

$$= (a+b) + d_a n - da + d_b n - db$$

$$= (a+b) + d_a(n-1) + d_b(n-1)$$

$$= (a+b) + (d_a + d_b)(n-1)$$

$$= \alpha + \beta(n-1) \text{ where } \alpha, \beta \text{ constants.}$$

changed

P68 [2.1]

P83 [8.3]\*

P71 [9.1] [9.2]

P71 [11]\*

P73 [4]

P74 [6.1]

P71

✓ [8.1]  $a=7, l=61, n=10$

$$S_{10} = \frac{10(7+61)}{2} = 340$$

✓ [8.2]  $a=-10, d=4, n=13$

$$S_{13} = \frac{13[(-20)+(12)4]}{2} = 182$$

[8.3]

changed

$$a=21, d=-6, l=-117$$

$$a_n = 21 - 6(n-1) = -117$$

$$\Rightarrow n = 24$$

$$S_{24} = \frac{24(21-117)}{2} = -1152$$

$$\therefore S_{24} = -1152$$

p71, ctd

[9.1] Prove  $1+2+3+\dots+n = \frac{n(n+1)}{2}$

Proof  $S_n = 1+2+3+\dots+n$

$$S_n = n+(n-1)+(n-2)+\dots+[n-kn-1]$$

$$2S_n = \underbrace{(n+1)+(n+1)+(n+1)+\dots+n+1}_{n\text{-terms}}$$

$$\therefore S_n = \frac{n(n+1)}{2}$$

[9.2] Prove  $1+3+5+\dots+(2n-1) = n^2$

Proof

$$S_n = 1+3+5+\dots+(2n-5)+(2n-3)+(2n-1)$$

$$S_n = (2n-1)+(2n-3)+\dots+5+3+1$$

$$\Rightarrow 2S_n = \underbrace{2n+2n+\dots+2n}_{n\text{ terms}}$$

$$= (2n)n$$

$$= 2n^2$$

$$\therefore S_n = n^2$$

✓ [10.1]  $S_{100} = \frac{100(101)}{2} = 5050$

✓ [10.2]  $S_{66} = 3+6+9+12+\dots+3n$   
 $= \frac{66(3+198)}{2}$

$$S_{66} = 6633$$

Alternative Proof

$$a=1, l=n$$

then

$$S_n = \frac{n(1+n)}{2}$$

□

Use Formula for  $S_n$   
Proved on page 70.

Alternative Proof

$$a=1, l=2n-1$$

then

$$S_n = \frac{n(1+2n-1)}{2}$$

$$= n^2$$

□

$$n \in \mathbb{Z}^+, 3n < 200$$
  
 $n < 198$

$$\frac{200}{3} = 66 \text{ R } 2$$

$$\text{so } 3 \cdot 66 = 198$$

Last mult of 3  
before 200

P 71, ctd

$$[ii] \quad S_n = 297, a = 45, d = -3; n = ?$$

$$297 = \frac{n}{2} [90 + (n-1)(-3)]$$

$$= \frac{n}{2} [90 - 3n + 3]$$

$$\Rightarrow n = 9 \text{ or } n = 22$$

$$S_9 = 297 \quad \underline{\text{or}} \quad S_{22} = 297$$

$$\therefore n = 22, n = 9$$

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END 3.1.1 - 3.1.2

p 73

$$\begin{aligned} \checkmark [1.1] \quad a_n &= ar^{n-1} \\ a_6 &= 1 \cdot 2^5 \\ a_6 &= 32 \end{aligned}$$

$$\checkmark [1.2] \quad a_5 = 3 \left(-\frac{1}{3}\right)^4 = 3 \left(\frac{1}{81}\right) = \frac{1}{27}$$

$$\checkmark [2.1] \quad a = \sqrt{2}, \quad r = \sqrt{2}, \quad a_n = \sqrt{2} (\sqrt{2})^{n-1}, \quad a_n = (\sqrt{2})^n$$

$$\checkmark [2.2] \quad a = 1, \quad r = -3, \quad a_n = 1 \cdot (-3)^{n-1}$$

$$\checkmark [2.3] \quad a = 1, \quad r = (-1), \quad a_n = 1 \cdot (-1)^{n-1}$$

$$\checkmark [3] \quad a_n = \frac{3^{n+1}}{2^n}$$

$$a_1 = \frac{3^2}{2^1} = \frac{9}{2}$$

$$r = \frac{3^{n+2}}{2^{n+1}} \cdot \frac{2^n}{3^{n+1}} = \frac{3}{2}$$

CHANGE

[4]

$$a_3 = 4, \quad a_5 = 36$$

$$a_5 = ar^4 = 36$$

$$a_3 = ar^2 = 4$$

$$\frac{a_5}{a_3} = \frac{ar^4}{ar^2} = \frac{36}{4}$$

$$r^2 = 9 \Rightarrow r = 3 \text{ or } r = -3$$

$$\text{then } 4 = 3^2 a \text{ or } 4 = (-3)^2 a$$

$$\text{so } a = \frac{4}{9}$$

$$\therefore a = \frac{4}{9}, \quad r = \pm 3$$

$$\checkmark [5.1] \quad a = 3, r = 2, n = 7, \text{ get } S_7$$

$$S_7 = \frac{3(1-2^7)}{1-2} = -3(1-2^7) = -3 + 3 \cdot 2^7$$

$$= -3 + 3 \cdot 2^7 = 381$$

$$\checkmark [5.2] \quad a = 1, r = -3, n = 6$$

$$S_6 = \frac{1 - (-3)^6}{1 + 3} = \frac{1}{4} (1 - (-3)^6) = -182$$

$$\checkmark [5.3] \quad a = 5, r = \frac{1}{2}, n = 5$$

$$S_5 = \frac{5(1 - (\frac{1}{2})^5)}{\frac{1}{2}} = 10(1 - \frac{1}{32}) = 10 \cdot \frac{31}{32} = \frac{310}{32} = \frac{155}{16}$$

$$\checkmark [6.1] \quad 13, 52, 208, 832, \dots$$

$$r = \frac{52}{13} = 4, \quad a = 13$$

$$S_n = \frac{13(1-4^n)}{3} = \frac{13[4^n - 1]}{3}$$

$$\checkmark [6.2] \quad \sqrt{3}, -1, \frac{1}{\sqrt{3}}, -\frac{1}{3}$$

$$r = -\frac{1}{\sqrt{3}}, \quad a = \sqrt{3}$$

$$S_n = \frac{\sqrt{3} [1 - (-\frac{\sqrt{3}}{3})^n]}{1 + \frac{\sqrt{3}}{3}}$$

p. 75

[7]

Find yearly payment  $a$  if

$A$  = amount of loan

$r$  = interest per year

$n$  = total number of years

Let  $y_k$  = balance remaining at end of year  $k$ .

$$y_1 = A(1+r) - a$$

$$y_2 = [A(1+r) - a](1+r) - a = A(1+r)^2 - a(1+r) - a$$

$$y_3 = [A(1+r)^2 - a(1+r) - a](1+r) - a = A(1+r)^3 - a(1+r)^2 - a(1+r) - a$$

⋮

$$y_n = A(1+r)^n - a(1+r)^{n-1} - a(1+r)^{n-2} - a(1+r)^{n-3} - \dots - a(1+r)^2 - a(1+r) - a$$

The loan is paid off when the balance becomes zero, thus

$$A(1+r)^n - a(1+r)^{n-1} - a(1+r)^{n-2} - a(1+r)^{n-3} - \dots - a(1+r)^2 - a(1+r) - a = 0$$

$$A(1+r)^n = a + a(1+r) + a(1+r)^2 + a(1+r)^3 + \dots + a(1+r)^{n-2} + a(1+r)^{n-1}$$

The RHS is a geometric series, so

$$A(1+r)^n = \frac{a[1-(r+1)^n]}{1-(1+r)}$$

Solving for  $a$ ,

$$a = \frac{Ar(r+1)^n}{1-(r+1)^n}$$

□

[8] This has been checked.

$$S_n = 1 \cdot 2 + 4 \cdot 2^2 + 7 \cdot 2^3 + \dots + [1 + 3(n-1)] 2^n$$

$$\frac{1}{2} S_n = 1 + 4 \cdot 2 + 7 \cdot 2^2 + 10 \cdot 2^3 + \dots + [1 + 3(n-1)] 2^{n-1}$$

$$\frac{1}{2} S_n - S_n = 1 + 3 \cdot 2 + 3 \cdot 2^2 + 3 \cdot 2^3 + \dots + 3 \cdot 2^{n-1} - [1 + 3(n-1)] 2^n$$

$$= 1 + 3 \underbrace{[2 + 2 \cdot 2 + 2 \cdot 2^2 + \dots + 2 \cdot 2^{n-2}]}_{\text{geom series}}$$

$a = 2$  and  $r = 2$ ,  $n-1$  terms

$$= 1 + 3 \left[ \frac{2(1-2^{n-1})}{1-2} \right] - [1 + 3(n-1)] 2^n$$

$$-\frac{1}{2} S_n = 1 - 3 [2(1-2^{n-1})] - [1 + 3(n-1)] 2^n$$

$$\Rightarrow S_n = -2 + 6 [2(1-2^{n-1})] + 2 \cdot 2^n [3n-2]$$

$$= -2 + 12 - 12 \cdot 2^{n-1} + 3n \cdot 2^{n+1} - 2^{n+2}$$

$$= 10 - 12 \cdot 2^{n-1} + 3n \cdot 2^{n+1} - 2^{n+2}$$

$$= 10 - 3 \cdot 2^{n+1} + 3n \cdot 2^{n+1} - 2^{n+2}$$

$$= 10 + 2^{n+1} (3n-3) - 2^{n+2}$$

$$= 10 + 3(n-1) 2^{n+1} - 2^{n+2}$$

$$= 10 + 3(n-1) 2^{n+1} - 2 \cdot 2^{n+1}$$

$$= 10 + [3(n-1) - 2] 2^{n+1}$$

$$= 10 + [3n-3-2] 2^{n+1}$$

$$= 10 + [3n-5] 2^{n+1}$$

$$\therefore S_n = 10 + [3n-5] \cdot 2^{n+1}$$

NOTE: when  $n=10$ ,  $S_{10} = 51210$ . If you compute  $S_{10}$  with your result and  $S_{10} \neq 51210$ , then your result is not EQUALLY TO this one.

P78,

[2] ctd

$$(n+1)^4 - 1^4 = 4[1^3 + 2^3 + \dots + n^3] + 6[1^2 + 2^2 + 3^2 + \dots + n^2] + 4[1 + 2 + 3 + \dots + n] + n$$

$$4[1^3 + 2^3 + \dots + n^3] = (n+1)^4 - 6[1^2 + 2^2 + 3^2 + \dots + n^2] - 4[1 + 2 + 3 + \dots + n] - (n+1)$$

$$= (n+1)^4 - 6 \left[ \frac{n(n+1)(2n+1)}{6} \right] - 4 \left[ \frac{n(n+1)}{2} \right] - (n+1)$$

$$= (n+1) \left[ (n+1)^3 - n(2n+1) - 2n - 1 \right]$$

$$= (n+1) (n^3 + 3n^2 + 3n + 1 - 2n^2 - n - 2n - 1)$$

$$= (n+1) (n^3 + n^2)$$

$$= (n+1)^2 \cdot n^2$$

$$\therefore [1^3 + 2^3 + 3^3 + \dots + n^3] = \left[ \frac{n(n+1)}{2} \right]^2$$

Then

$$[6^3 + 7^3 + 8^3 + \dots + 13^3] = \left( \frac{13(14)}{2} \right)^2 - \left( \frac{5(6)}{2} \right)^2$$

$$= 8056$$

p 81

$$[3.1] \sum_{i=1}^n i^3$$

$$[3.2] \sum_{k=1}^{10} [5 + 4(k-1)]$$

$$5 + 4k - 4 = 4k + 1$$

$$4k + 1 = 41 \Rightarrow 4k = 40 \Rightarrow k = 10$$

$$[4.1] \sum_{k=1}^8 (3k+1) = 4 + 7 + 10 + 13 + 16 + 19 + 22 + 25$$

$$[4.2] \sum_{i=0}^4 \left(\frac{1}{2}\right)^i = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$$

$$[4.3] \sum_{j=1}^n j(j+1) = 2 + 6 + 12 + \dots + n(n+1)$$

p 81

$$\begin{aligned} [5.1] \sum_{k=1}^n (5k+1) &= 5 \sum_{k=1}^n k + \sum_{k=1}^n 1 \\ &= 5 \left[ \frac{n(n+1)}{2} \right] + n \\ &= \frac{5n(n+1)}{2} + \frac{2n}{2} \\ &= \frac{5n^2 + 5n + 2n}{2} \\ &= \frac{1}{2} [5n^2 + 7n] \\ &= \frac{1}{2} n [5n + 7] \end{aligned}$$

P 81

$$[5.2] \sum_{i=1}^{n-1} (i+1)(i-2)$$

$$= \sum_{i=1}^{n-1} i^2 - i - 2$$

$$= \sum_{i=1}^{n-1} i^2 - \sum_{i=1}^{n-1} i - \sum_{i=1}^{n-1} 2$$

$$= \frac{(n-1)(n)(2n-1)}{6} - \frac{(n-1)(n)}{2} - 2(n-1)$$

$$= \frac{n-1}{6} [n(2n-1) - 3n - 12]$$

$$= \frac{n-1}{6} [2n^2 - n - 3n - 12]$$

$$= \frac{n-1}{6} (2n^2 - 4n - 12)$$

$$= \frac{n-1}{3} (n^2 - 2n - 6)$$

$$= \frac{(n-1)(n^2 - 2n - 6)}{3}$$

CHECKED.

4-24

$$[6.1] \quad 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n(n+1)$$

$$= \sum_{i=1}^n i(i+1) = \sum_{i=1}^n i^2 + i = \sum_{i=1}^n i^2 + \sum_{i=1}^n i$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n+1}{6} [n(2n+1) + 3n]$$

$$= \frac{n+1}{6} [2n^2 + n + 3n]$$

$$= \frac{n+1}{3} [n^2 + 2n]$$

$$= \frac{n(n+1)(n+2)}{3}$$

$$[6.2] = 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + n(n+1)(n+2)$$

$$= \sum_{k=1}^n k(k+1)(k+2) = \sum_{k=1}^n k^3 + 3k^2 + 2k$$

$$= \frac{n^2(n+1)^2}{4} + 3 \left[ \frac{n(n+1)(2n+1)}{6} \right] + 2 \left[ \frac{n(n+1)}{2} \right]$$

$$= \frac{1}{4} n(n+1) [n(n+1) + 2(2n+1) + 4]$$

$$= \frac{1}{4} n(n+1) [n^2 + n + 4n + 2 + 4]$$

$$= \frac{1}{4} n(n+1) [n^2 + 5n + 6]$$

$$= \frac{1}{4} n(n+1)(n+2)(n+3)$$

[7.1]

$$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \dots + n(n+2)$$

$$\sum_{k=1}^n k(k+2) = \sum_{k=1}^n k^2 + 2k = \sum_{k=1}^n k^2 + \sum_{k=1}^n 2k$$

$$= \frac{n(n+1)(2n+1)}{6} + n(n+1)$$

$$= \frac{1}{6} [n(n+1)(2n+1) + 6n(n+1)]$$

$$= \frac{n+1}{6} [2n^2 + n + 6n]$$

$$= \frac{n+1}{6} [2n^2 + 7n]$$

$$= \frac{n(n+1)(n+7)}{6}$$

$$[7.2] \quad 1^2 \cdot 2 + 2^2 \cdot 5 + 3^2 \cdot 8 + 4^2 \cdot 11 + 5^2 \cdot 14 + \dots + n^2(2+3(n-1))$$

$$= \sum_{k=1}^n k^2(3k-1)$$

$$= \sum_{k=1}^n 3k^3 - k^2 = 3 \sum_{k=1}^n k^3 - \sum_{k=1}^n k^2$$

$$= 3 \left[ \frac{n(n+1)}{2} \right]^2 - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{3n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{12} [9n^2(n+1)^2 - 2n(n+1)(2n+1)]$$

$$= \frac{n(n+1)}{12} [9n(n+1) - 2(2n+1)]$$

NEXT  $\rightarrow$

P81

[7.2] ctd

$$= \frac{n(n+1)}{2} [9n^2 + 9n - 4n - 2]$$

$$= \frac{n(n+1)}{2} [9n^2 + 5n - 2]$$

checked ✓ 16 4(9)(11)

[8.1]

$$1, 2, 5, 10, 17, 26, \dots$$

$$\begin{array}{ccccccc} \swarrow & \searrow & \swarrow & \searrow & \swarrow & \searrow & \swarrow \\ 1 & 3 & 5 & 7 & 9 & \dots & \end{array} = \{b_k\}$$

$$a_n = a_1 + \sum_{k=1}^{n-1} b_k$$

$$= 1 + \sum_{k=1}^{n-1} [1 + 2(k-1)]$$

$$= 1 + \sum_{k=1}^{n-1} 1 + 2 \sum_{k=1}^{n-1} k - 2 \sum_{k=1}^{n-1} 1$$

$$= 1 + (n-1) + (n-1)n - 2(n-1)$$

$$= 1 + n - 1 + n^2 + n - 2n + 2$$

$$\boxed{a_n = n^2 - 2n + 2}$$

$$\sum_{k=1}^n k^2 - 2k + 2 = \frac{n(n+1)(2n+1)}{6} - n(n+1) + 2n$$

$$= \frac{n}{6} [(n+1)(2n+1) - 6(n+1) + 12]$$

$$= \frac{n}{6} [2n^2 + 3n + 1 - 6n - 6 + 12]$$

$$\boxed{\sum_{k=1}^n a_k = \frac{n}{6} [2n^2 - 3n + 7]}$$

[8.2]

$$\{a_n\} = 3, 4, 1, 10, -17, 64, \dots$$

$$\{b_n\} = 1, -3, 9, -27, 81, \dots$$

$$\begin{aligned} a_n &= 3 + \sum_{k=1}^{n-1} [-3]^{k-1} \\ &= 3 + \frac{1 - [-3]^{n-1}}{1 - [-3]} \\ &= 3 + \frac{1 - [-3]^{n-1}}{4} \end{aligned}$$

$[-3]^{k-1}$  is geom series  $a=1, r=-3$   
So sum of 1st  $n-1$  terms of  
this series

$$a_n = \frac{13 - [-3]^{n-1}}{4}$$

$$\sum_{k=1}^n a_n = \sum_{k=1}^n \frac{13 - [-3]^{k-1}}{4}$$

$$= \frac{13n}{4} - \frac{1}{4} \left[ \frac{1 - [-3]^n}{4} \right]$$

$$\sum a_n = \frac{13n}{4} - \frac{1}{16} [1 - [-3]^n]$$

$[-3]^{k-1}$  geom series  
and desire sum of  
1st  $n$  terms.

NEED TO NOT get confused about index

p. 83 [9.1]

$$\sum_{k=1}^n \left( \frac{-1}{k(k+2)} \right) = \sum_{k=1}^n \frac{-1}{2} \left( \frac{1}{k} - \frac{1}{k+2} \right)$$

$$= \frac{-1}{2} \sum_{k=1}^n \left( \frac{1}{k} - \frac{1}{k+2} \right)$$

$$= \frac{-1}{2} \left\{ \left(1 - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{4} - \frac{1}{6}\right) + \dots + \left(\frac{1}{n-5} - \frac{1}{n-3}\right) + \right. \\ \left. \left(\frac{1}{n-4} - \frac{1}{n-2}\right) + \left(\frac{1}{n-3} - \frac{1}{n-1}\right) + \left(\frac{1}{n-2} - \frac{1}{n}\right) + \left(\frac{1}{n-1} - \frac{1}{n+1}\right) + \left(\frac{1}{n} - \frac{1}{n+2}\right) \right\}$$

$$= \frac{-1}{2} \left( 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} \right)$$

$$= \left( \frac{-3n^2 - 5n}{4(n+1)(n+2)} \right)$$

p. 83 [9.2]

$$\begin{aligned}\sum_{k=1}^n \frac{1}{(2k)^2-1} &= \sum_{k=1}^n \left( \frac{1}{2(2k-1)} - \frac{1}{2(2k+1)} \right) \\ &= \frac{1}{2} \sum_{k=1}^n \left( \frac{1}{2k-1} - \frac{1}{2k+1} \right) \\ &= \frac{1}{2} \left\{ \left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \dots + \left(\frac{1}{2(n-3)-1} - \frac{1}{2(n-3)+1}\right) + \right. \\ &\quad \left. \left(\frac{1}{2(n-2)-1} - \frac{1}{2(n-2)+1}\right) + \left(\frac{1}{2(n-1)-1} - \frac{1}{2(n-1)+1}\right) + \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) \right\} \\ &= \frac{1}{2} \left\{ \left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{7}\right) + \dots + \right. \\ &\quad \left. \left(\frac{1}{2n-7} - \frac{1}{2n-5}\right) + \left(\frac{1}{2n-5} - \frac{1}{2n-3}\right) + \left(\frac{1}{2n-3} - \frac{1}{2n-1}\right) + \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right) \right\} \\ &= \frac{1}{2} \left(1 - \frac{1}{2n+1}\right) \\ &= \frac{n}{2n+1}\end{aligned}$$

[10]

$$\begin{aligned}\sum_{k=1}^n \frac{1}{k(k+1)(k+2)} &= \frac{1}{2} \sum_{k=1}^n \left( \frac{1}{k(k+1)} - \frac{1}{(k+1)(k+2)} \right) \\ &= \frac{1}{2} \left\{ \left( \frac{1}{2} - \frac{1}{6} \right) + \left( \frac{1}{6} - \frac{1}{12} \right) + \left( \frac{1}{12} - \frac{1}{20} \right) + \dots + \left( \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right) \right\} \\ &= \frac{1}{2} \left( \frac{1}{2} - \frac{1}{(n+1)(n+2)} \right) \\ &= \left( \frac{n^2+3n}{4(n+1)(n+2)} \right)\end{aligned}$$

$$[11] \quad S_n = n^3 - n + 2.$$

$$a_n = S_n - S_{n-1}$$

$$= n^3 - n + 2 - [(n-1)^3 - (n-1) + 2]$$

$$= n^3 - n + 2 - [n^3 - 3n^2 + 3n - 1 - n + 1 + 2]$$

$$= n^3 - n + 2 - n^3 + 3n^2 - 3n + 1 + n - 1 - 2$$

$$= 3n^2 - 3n$$

$$= 3n(n-1)$$

[07-08-25-T11]  
Answers to J11-BA page 85 Exercises

*These have been checked*

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### J11 Exercises p 85

[1]

**Solution A.** Since  $S_3 = S_5$ ,  $a_1 + a_2 + a_3 = a_1 + a_2 + a_3 + a_4 + a_5$ . But this means  $(a_4 + a_5)$  must contribute nothing to the sum. Thus,  $a_4 + a_5 = 0 \implies (5 + 3d) + (5 + 4d) = 0 \implies 7d = -10 \implies d = \frac{-10}{7}$ .

**Solution B.** Use  $S_n = \frac{n}{2}[2a + (n-1)d]$ . Thus,  $S_3 = \frac{3}{2}[10 + 2d] = \frac{5}{2}[10 + 4d] = S_5 \implies d = \frac{-10}{7}$ .

[2.1]

$$a_5 = a_1 + 4d = 108$$

$$a_{20} = a_1 + 19d = -237$$

Then,  $a_1 = 200$ ,  $d = -23$

[2.2]

$$S_n = \frac{n[2 \cdot 200 - 23(n-1)]}{2} = \frac{1}{2}(423n - 23n^2) = \frac{n}{2}(423 - 23n)$$

The roots of  $y = \frac{x}{2}(423 - 23x)$  are  $x = 0$  and  $x = \frac{423}{23}$ . The extreme value, in this case its maximum,

occurs at  $x = \frac{1}{2} \left( \frac{423}{23} \right) = 9 \frac{9}{46}$ . Therefore,  $n = 9$ . [Note: since  $9 \frac{9}{46}$  is closer to 9 than to 10, so the maximum of  $S_n$  is at 9, not 10.]

[3]

$$a_3 = ar^2 = 12$$

$$a_6 = ar^5 = 96$$

Solving simultaneously,  $\frac{12}{r^2}(r^5) = 96 \implies r^3 = 8 \implies r = 2$ .  $a = \frac{12}{4} = 3$ . So,  $a_n = 3 \cdot 2^{n-1}$ . Then,

[3.1]

$$\sum_{k=1}^n (3 \cdot 2^{k-1})^2 = \sum_{k=1}^n 9 \cdot 2^{2(k-1)} = \sum_{k=1}^n 9 \cdot 4^{(k-1)}$$

This is the sum of a geometric series with  $a = 9$  and  $r = 4$ . Thus, the sum of the first  $n$  terms is

$$S_n = 9 \left( \frac{4^n - 1}{4 - 1} \right) = 3(4^n - 1)$$

[3.2]

Product of first  $n$  terms:

$$\begin{aligned} & \prod_{i=1}^n (3 \times 2^{i-1}) \\ &= [(3 \times 2^0) (3 \times 2^1) (3 \times 2^2) (3 \times 2^3) \times \dots \times (3 \times 2^{n-1})] \\ &= 3^n (2^1 \times 2^2 \times 2^3 \times \dots \times 2^{n-1}) \\ &= 3^n 2^{[1+2+3+\dots+(n-1)]} \\ &= 3^n 2^{\frac{n(n-1)}{2}} \end{aligned}$$

[4.1]

$$\begin{aligned} \sum_{i=1}^n (1 + 3(n-1))^2 &= \sum_{i=1}^n (4 - 12i + 9i^2) = \sum_{i=1}^n 4 - \sum_{i=1}^n 12i + \sum_{i=1}^n 9i^2 \\ &= 4n - 12 \left( \frac{n(n+1)}{2} \right) + 9 \left( \frac{n(n+1)(2n+1)}{6} \right) \\ &= \frac{n}{2} (6n^2 - 3n - 1) \end{aligned}$$

[4.2]

$$\begin{aligned} \sum_{i=1}^n (2i)(3 + (i-1))^2 &= \sum_{i=1}^n (8i + 8i^2 + 2i^3) = 2 \sum_{i=1}^n i^3 + 8 \sum_{i=1}^n i^2 + 8 \sum_{i=1}^n i \\ &= 2 \left( \frac{n(n+1)}{2} \right)^2 + 8 \left( \frac{n(n+1)(2n+1)}{6} \right) + 8 \left( \frac{n(n+1)}{2} \right) \\ &= \frac{n}{6} (32 + 51n + 22n^2 + 3n^3) \end{aligned}$$

[5]

$$\{a_n\} = 2, 3, 9, 18, 28, 37, 43, 44, \dots$$

$$\{b_n\} = 1, 6, 9, 10, 9, 6, 1, \dots$$

$$\{c_n\} = 5, 3, 1, -1, -3, -5, \dots$$

$$c_n = 5 - 2(n - 1)$$

$$\begin{aligned} b_n &= 1 + \sum_{k=1}^{n-1} (5 - 2(k - 1)) \\ &= 1 + \sum_{k=1}^{n-1} (7 - 2k) \\ &= (1 + 7(n - 1) - n(n - 1)) \end{aligned}$$

$$\therefore b_n = -6 + 8n - n^2$$

$$a_n = 2 + \sum_{k=1}^{n-1} (-6 + 8k - k^2)$$

$$\therefore a_n = \frac{1}{6} (48 - 61n + 27n^2 - 2n^3)$$

[6]

Let  $S_n$  be the sum of the first  $n$  terms of  $\{a_n\}$ . If  $S_n = an^2 + bn + c$ , then is  $\{a_n\}$  an arithmetic sequence?

Suppose  $S_n = an^2 + bn + c$ . Then  $S_{n-1} = a(n-1)^2 + b(n-1) + c$ . Thus,

$$a_n = S_n - S_{n-1} = an^2 + bn + c - (a(n-1)^2 + b(n-1) + c)$$

$$a_n = (b - a) + (2a)n$$

$\therefore$  This is an arithmetic sequence with first term  $(b - a)$  and common difference  $2a$ . That is,

$$\{(b - a) + (n - 1)(2a)\}$$