[11-05-11-T8]

Completing the square

- **Completing the square. This is a convienent process that always works the same way.**

EXAMPLE 1.

$$x^2 + 8x + 5 = 12$$

Step 1: Constants to the RHS, terms with the unknown to the LHS.

$$x^2 + 8x = 7$$

Step 2: Add the square of one-half the coefficient of x to both sides of the equation. In this example, the coefficient of x is 8, half of 8 is 4, and 4^2 is 16. Thus,

$$x^2 + 8x + 16 = 23$$

The LHS is now a perfect square trinomial. As such it will factor using the form $(a+b)^2 = a^2 + 2ab + b^2$. In the above example, $x^2 + 8x + 16 = 23 \Longleftrightarrow (x+4)^2 = 23$.

EXAMPLE 2.

$$x^2 - 8x + 5 = 12$$

Step 1: Constants to the RHS, terms with the unknown to the LHS.

$$x^2 - 8x = 7$$

Step 2: Add the square of one-half the coefficient of x to both sides of the equation. In this example, the coefficient of x is -8, half of -8 is -4, and $(-4)^2$ is 16. Thus,

$$x^2 - 8x + 16 = 23$$

The LHS is now a perfect square trinomial. As such it will factor using the form $(a-b)^2 = a^2 - 2ab + b^2$. In the above example, $x^2 - 8x + 16 = 23 \Longleftrightarrow (x-4)^2 = 23$.

■ **For each equation, solve for x by completing the square.**

[1] $x^2 + 8x + 5 = 10$

[2] $a^2 + 6a + 5 = 8$

[3] $x^2 + 6x - 8 = 1$

[4] $x^2 - 2x - 12 = 0$

[5] $x^2 + 10x + 1 = 3$

[6] $x^2 - 2x - 8 = -2$

[7] $x^2 - 4x = 7$

[8] $x^2 + x + 12 = 3$

[9] $x^2 - \frac{2}{3}x - \frac{5}{9} = 0$

[10] $x^2 - 10x + 3 = 6$

Answers

[1] $x^2 + 8x + 5 = 10 \Longleftrightarrow x^2 + 8x + 16 = 21 \Longleftrightarrow (x+4)^2 = 21 \Longrightarrow x = -4 \pm \sqrt{21}$

[2] $a^2 + 6a + 5 = 8 \Longleftrightarrow x^2 + 8x + 9 = 12 \Longleftrightarrow (x+4)^2 = 12 \Longrightarrow x = -4 \pm 2\sqrt{3}$

[3] $x^2 + 6x - 8 = 1 \Longleftrightarrow x^2 + 6x + 9 = 18 \Longleftrightarrow (x+3)^2 = 18 \Longrightarrow x = -3 \pm 3\sqrt{2}$

[4] $x^2 - 2x - 12 = 0 \Longleftrightarrow x^2 - 2x + 1 = 13 \Longleftrightarrow (x+1)^2 = 13 \Longrightarrow x = -1 \pm \sqrt{13}$

[5] $x^2 + 10x + 1 = 3 \Longleftrightarrow x^2 + 10x + 25 = 27 \Longleftrightarrow (x+5)^2 = 27 \Longrightarrow x = -5 \pm 3\sqrt{3}$

[6] $x^2 - 2x - 8 = -2 \Longleftrightarrow x^2 - 2x + 1 = 7 \Longleftrightarrow (x-1)^2 = 7 \Longrightarrow x = 1 \pm \sqrt{7}$

[7] $x^2 - 4x = 7 \Longleftrightarrow x^2 - 4x + 4 = 11 \Longleftrightarrow (x-2)^2 = 11 \Longrightarrow x = 2 \pm \sqrt{11}$

[8] $x^2 + x + 12 = 3 \Longleftrightarrow x^2 + x + \frac{1}{4} = -9 + \frac{1}{4} \Longleftrightarrow \left(x + \frac{1}{2}\right)^2 = \frac{-35}{4} \Longrightarrow$ no real roots

[9] $x^2 - \frac{2}{3}x - \frac{5}{9} = 0 \Longleftrightarrow x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{6}{9} \Longleftrightarrow \left(x - \frac{1}{3}\right)^2 = \frac{2}{3} \Longrightarrow x = \frac{1}{3} \pm \sqrt{\frac{2}{3}}$

[10] $x^2 - 10x + 3 = 6 \Longleftrightarrow x^2 - 10x + 25 = 28 \Longleftrightarrow (x-5)^2 = 28 \Longrightarrow x = 5 \pm 2\sqrt{7}$