[09-12-10-T8]

Study Guide Equation of Line

Please note that (1) the word "line" as used here means a straight line and (2) certain topics and formulae are excluded by the use of the word "**NO**" preceeding the topic number; #16, 17, 18 are excluded as are the formulae for the midpoint and length of a line segment.

■ After studying the linear function in grade 8, you will be expected to do the following

- 1. Find the slope of a line given two points on the line.
- 2. Find the slope of a line given the equation of the line.
- 3. Graph a line given the equation of the line.
- 4. Graph a line given its slope and a point on the line.
- 5. Use slope to decide whether two lines are parallel or perpendicular.
- 6. Determine whether three points lie on a line.
- 7. Find the slope and y-intercept of a line given its equation.
- 8. Know and apply the three forms of the linear equation.
- 9. Write the equation of a line given its slope and y-intercept.
- 10. Write the equation of a line given its slope and a point on the line.
- 11. Write the equations of lines that are horizontal or vertical.
- 12. Write the equation of a line given two points on the line.
- 13. Write the equation of a line through a given point and parallel or perpendicular to a given line.
- 14. Determine the point of intersection of two lines given the equation for each of the lines.
- 15. Find the slope, the x-intercept, and the y-intercept of a line whose equation is given in any of the three forms. This includes the special cases of a vertical or a horizontal line.
- **NO** 16. Determine the midpoint a line segment.
- **NO** 17. Determine the length a line segment.
- **NO** 18. Find the perpendicular bisector of a line segment.

- **Famous formulae**

Standard form $a x + b y + c = 0$

Slope-intercept form $y = m x + b$

Point-slope form $y - y_1 = m(x - x_1)$

Slope of line $\frac{y_2 - y_1}{x_2 - x_1}$

Slopes of perpendicular lines $m_1 m_2 = -1$

NO Midpoint $\bar{x} = \frac{x_1 + x_2}{2}$ and $\overline{y} = \frac{y_1 + y_2}{2}$.

NO Length $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note: this reference section will not appear on the exam.

Please note

The only way to master this material is to understand the basic ideas and apply them to specific problems. Typically, a sketch will help you ***see*** which ideas apply and how they might be used. The habit of sketching functions (and the ability to do so) will benefit you greatly at higher course levels in mathematics.

Typical problems and their solutions

- 1. Find the slope of a line through $P(-1, 3)$, $Q(4, 7)$.

Solution:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{4+1} = \frac{4}{5}.$$

- 2. Find the slope of the line whose equation is $3x + 2y = 7$.

Solution:

$$3x + 2y = 7 \Longleftrightarrow y = \frac{-3}{2}x + \frac{7}{2}$$

From which we see the slope is $\frac{-3}{2}$.

- 3a. Graph a line whose equation is $3x + 2y = 7$.

Solution:

Substitute $x = 0$ to obtain point $P(0, \frac{7}{2})$ and substitute $y = 0$ to obtain point $Q(\frac{7}{3}, 0)$. Plot points P and Q, then draw a line through them.

- 3b. Graph a line whose equation is $y = -\frac{3}{2}x + \frac{7}{2}$.

Solution:

Note that the slope is $\frac{-3}{2}$ and that the y-intercept is $\frac{7}{2}$ (hence the point $(0, \frac{7}{2})$ is on the line. Then proceed as in #4 below.

- 4. Graph a line whose slope is $\frac{-3}{2}$ that passes through the point $P(0, \frac{7}{2})$.

Solution:

Plot the point $P(0, \frac{7}{2})$. Use the slope to plot a second point *3* units down from point P and 2 units to the right of point P.

- 5a. Let line l_1 be given by $y = 3x - 2$ and line l_2 be given by $y = \frac{-1}{3}x + 1$. Is l_1 parallel to l_2?.

Solution:

No, because $m_1 = 3$, $m_2 = \frac{-1}{3}$, $m_1 \neq m_2$.

- 5b. Let line l_1 be given by $y = 3x - 2$ and line l_2 be given by $y = \frac{-1}{3}x + 1$. Is l_1 perpendicular to l_2?.

Solution:

Yes, because $m_1 = 3$, $m_2 = \frac{-1}{3}$, $m_1 m_2 = -1$.

- 6. Do the points $P(-2, -3)$, $Q(2, 3)$, $R(14, 21)$ lie on a single line?.

Solution:

Yes. The line through points P and Q has slope $m_{\text{PQ}} = \frac{3+3}{2+2} = \frac{3}{2}$ and the line through points Q and R has slope $m_{\text{QR}} = \frac{21-3}{14-2} = \frac{18}{12} = \frac{3}{2}$. Since both lines pass through Q and have the same slope, they are the same line.

- 7. Find the slope and y-intercept of the line $-5x + 3y + 2 = 0$.

Solution:

$-5x + 3y + 2 = 0 \Longleftrightarrow y = \frac{5}{3}x - \frac{2}{3}$. Thus the line's slope is $\frac{5}{3}$ and its y-intercept is $\frac{-2}{3}$.

- 8a. Write in **point-slope form** the equation of a line that includes points $P(0, 3)$, $Q(5, 7)$.

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x - 0)$

$m = \frac{7-3}{5-0} = \frac{4}{5}$

$\therefore\ y - 3 = \frac{4}{5}(x - 0)$

- 8b. Write in **slope-intercept form** the equation of a line that includes points $P(-1, 3)$, $Q(5, 7)$.

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x + 1)$

$m = \frac{7-3}{5+1} = \frac{4}{6} = \frac{2}{3}$

$y - 3 = \frac{2}{3}(x + 1)$ (*The equation in point-slope form.*)

$y = \frac{2}{3}x + \frac{2}{3} + 3$

$\therefore\ y = \frac{2}{3}x + \frac{11}{3}$ (*The equation in slope-intercept form.*)

- 8c. Write in **standard form** the equation of a line that includes points $P(-1, 3)$, $Q(5, 7)$.

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x + 1)$

$m = \frac{7-3}{5+1} = \frac{4}{6} = \frac{2}{3}$

$y - 3 = \frac{2}{3}(x + 1)$ (*The equation in point-slope form.*)

$3y - 9 = 2(x + 1)$

$\therefore\ -2x + 3y = 11$ (*The equation in standard form.*)

- 9. Write the equation of a line whose slope is *2* and which crosses the y-axis at $y = -\frac{3}{5}$. Answer in **slope-intercept** form.

Solution:

Given a slope and an intercept, we typically use the slope-intercept form of the linear equation to find the equation of the line. Thus,

$y = m x + b$

$\therefore \; y = 2x - \frac{3}{5}$

- 10. Write the equation of a line whose slope is *2* and which passes through $P(-2, \frac{-7}{3})$. Answer in **slope-intercept** form.

Solution:

Given a slope and a point, we typically use the point-slope form of the linear equation to find the equation of the line. Thus,

$y - y_1 = m(x - x_1)$

$y - (-\frac{7}{3}) = 2(x - (-2))$

$y + \frac{7}{3} = 2(x + 2)$ (*The equation in point-slope form.*)

$y + \frac{7}{3} = 2x + 4$

$\therefore \; y = 2x + \frac{5}{3}$ (*The equation in slope-intercept form.*)

- 11a. Write the equation of the line through $P(-4, 2)$ parallel to the y-axis.

Solution:

The x-coordinate of every point on this line is -4. The y-coordinate takes all values. Therefore,

$x = -4$ is the equation of this line.

- 11b. Write the equation of the line through $P(-4, 2)$ parallel to the x-axis.

Solution:

The y-coordinate of every point on this line is 2. The x-coordinate takes all values. Therefore,

$y = 2$ is the equation of this line.

- 12. Write the equation of the through $P(2, 3)$, $Q(5, 11)$. Answer in **slope-intercept** form

Solution:

Given two points, we typically use the point-slope form of the linear equation to find the equation of the line through the points. Thus,

$y - y_1 = m(x - x_1)$

$y - 3 = m(x - 2)$

$m = \frac{11-3}{5-2} = \frac{8}{3}$

$y - 3 = \frac{8}{3}(x - 2)$ (*The equation in point-slope form.*)

$\therefore\ y - 3 = \frac{8}{3}x + \frac{25}{3}$ (*The equation in slope-intercept form.*)

- 13. Write the equation of a line l_2 through point $P(-7, 3)$ and parallel to $l_1 :\ 3x + 11y = 4$. Answer in **standard form**.

We are given a point. The slope must match that of l_1, since the lines are parallel. Our initial equation for l_1 will be in point-intercept form. Then we will rewrite the equation in standard form.

$y - y_1 = m(x - x_1)$

$y - 3 = m(x + 7)$

$3x + 11y = 4 \Longleftrightarrow y = \frac{-3}{11}x + \frac{4}{11}$, so the slope is $-\frac{3}{11}$.

$y - 3 = \frac{-3}{11}(x + 7)$

$11y - 33 = -3(x + 7)$

$\therefore\ 3x + 11y = 12$

- 14a. Determine the point of intersection lines l_1 and l_2 whose equations are respectively $y = 2x - 4$ and $y = 10x - 12$.

Solution:

The point of intersection is a point common to both lines. The coordinates of the point of intersection must satisfy both equations. Finding these coordinates amounts to solving the pair of simultaneous equations

$$\begin{pmatrix} -2x + y = -4 \\ -10x + y = -12 \end{pmatrix}$$

The solution of this pair of equations is $(1, -2)$. Therefore, the lines intersect at the point $P(1, -2)$.

- 14b. Determine the point of intersection lines l_1 and l_2 whose equations are respectively $y = 2x - 4$ and $y = 2x - 12$.

  Solution:

  It is obvious from the equations that the lines are parallel. Thus, they do not intersect. The answer to this question is "The point of intersection does not exist".

- 15a. Find the slope, the x-intercept, and the y-intercept of a line whose equation is $y = 2x - 7$.

  Solution:

  By inspection, the slope is *2*.

  Since the line crosses the x-axis at $y = 0$, the x-intercept is found by substituting 0 for y in the equation. Doing this, we find that $0 = 2x - 7 \Longrightarrow x = \frac{7}{2}$. Hence, the x-intercept is $\frac{7}{2}$.

  Since the line crosses the y-axis at $x = 0$, the y-intercept is found by substituting 0 for x in the equation. Doing this, we find that $y = 2(0) - 7 \Longrightarrow y = -7$. Hence, the y-intercept is -7.

- 15b. Find the slope, the x-intercept, and the y-intercept of a line whose equation is $y = -2$.

  Solution:

  The line is horizontal, thus it has a slope equal to zero..

  Since the line is horizontal, it is parallel to the x-axis. Hence, it never crosses the x-axis. Therefore, the x-intercept does not exist.

  Since the y-coordinate of the line is a constant, *2*, the line crosses the y-axis at $y = 2$. Therefore, the y-intercept is *2*.

- 15c. Find the slope, the x-intercept, and the y-intercept of a line whose equation is $x = -7$.

  Solution:

  The line is vertical, thus its slope is undefined.

  Since the line is vertical, it is parallel to the y-axis. Hence, it never crosses the y-axis. Therefore, the y-intercept does not exist.

  Since the x-coordinate of the line is a constant, -7, the line crosses the x-axis at $x = -7$. Therefore, the x-intercept is -7.

- **NO** 16. Determine the midpoint, M, a the line segment $\overline{PQ}$, $P(-3, 5)$, $Q(8, -12)$.

Solution:

The coordinates at the midpoint of line segment $P(x_1, y_1)$, $Q(x_2\ y_2)$ are

$$\bar{x} = \frac{x_1+x_2}{2} \text{ and } \bar{y} = \frac{y_1+y_2}{2}.$$

$\bar{x} = \frac{-3+8}{2} = \frac{5}{5}$, $\bar{y} = \frac{5-12}{2} = \frac{-7}{2}$.

Thus, $M(\bar{x}, \bar{y}) = (\frac{5}{2}, -\frac{7}{2})$.

- **NO** 17. Determine the length a line segment $\overline{PQ}$, $P(-3, 5)$, $Q(8, -12)$.

Solution:

The length ℓ of line segment $P(x_1, y_1)$, $Q(x_2\ y_2)$ is

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\text{. So,}$$

$l = \sqrt{(8-3)^2 + (-12-5)^2} = \sqrt{25 + 289} = (\sqrt{314})$. This is approximately 17.75.

- **NO** 18. Find the perpendicular bisector of a line segment $\overline{PQ}$, $P(-3, 5)$, $Q(9, -13)$.

Solution:

The form $y - y_1 = m(x - x_1)$ is handy.

The perpendicular bisector of $\overline{PQ}$ must go through the midpoint $M(x_1, y_1)$ of $\overline{PQ}$.

$$\overline{x_1} = \frac{-3+9}{2} = 3,\ \bar{y} = \frac{5-13}{2} = -4\text{, so } M(x_1, y_1) = M(3, -4).$$

The slope m_{PQ} of $\overline{PQ}$ is

$$m_{PQ} = \frac{-13-5}{9+3} = -\frac{3}{2},$$

so the slope m of the line perpendicular to $\overline{PQ}$ is

$$m(\frac{-3}{2}) = -1 \Longrightarrow m = \frac{2}{3}.$$

Thus, the point-slope equation of the perpendicular bisector of $\overline{PQ}$ is

$$y + 4 = \frac{2}{3}(x - 3)$$