Using Cues in a Problem

The sentences that must be translated from words into algebraic symbols are often not stated fully in the problem but instead are suggested by words of the problem. Thus, in Example 1 below, the word "perimeter" is the cue which causes us to recall that, in a rectangle, twice the length plus twice the width equals the perimeter. It is the words underscored in the preceding sentence that we translate into the symbols $5w + 2w = 140$ in Step 4 of that example.

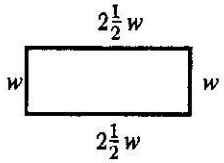
Example 1. The length of a rectangular table is $2\frac{1}{2}$ times its width. The perimeter of the rectangle is 140 inches. Find the length and width.

Solution: Since the problem deals with a geometric figure,

STEP 1. Draw the figure.

STEP 2. Let w = the number of inches in the width. (After drawing the figure re-read the problem to decide which quantity it is easier to represent by a letter.)

STEP 3. Then $2\frac{1}{2}w$ = the number of inches in the length. (Now put these symbols on the drawing.)



STEP 4. The "perimeter" is the length of the boundary. In a rectangle, the perimeter is twice the length plus twice the width. Hence

$$5w + 2w = 140$$

Then

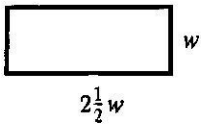
$$7w = 140$$
$$w = 20$$
$$2\tfrac{1}{2}w = 50$$

$\therefore$ The length is 50 in. and the width 20 in.

NOTE. The symbol $\therefore$ stands for "therefore."

A complete and satisfactory solution would look like this:

Let w = the no. of inches in the width.
Then $2\frac{1}{2}w$ = the no. of inches in the length.



$$\therefore 2w + 5w = 140$$
$$7w = 140$$
$$w = 20$$
$$2\tfrac{1}{2}w = 50$$

$\therefore$ The length is 50 in. and the width is 20 in.

Check: 50 is $2\frac{1}{2}$ times 20;
the perimeter is $2 \times 50 + 2 \times 20 = 100 + 40 = 140$.
Hence 50 in. and 20 in. are dimensions which satisfy the conditions of the problem.

Example 2. Joe received \$3.50 for cutting and trimming a lawn. He was paid in half-dollars, quarters, and dimes. The number of quarters was twice the number of half-dollars, the number of dimes was $\frac{5}{3}$ the number of half-dollars. How many coins were there?

Solution: Let h = the number of half-dollars
Then $2h$ = the number of quarters
and $\frac{5}{3}h$ = the number of dimes
$50h$ = the value in cents of h half-dollars
$50h$ = the value in cents of $2h$ quarters
$\frac{50}{3}h$ = the value in cents of $\frac{5}{3}h$ dimes

Then $$50h + 50h + \frac{50h}{3} = 350$$

$$h + h + \frac{h}{3} = 7 \qquad \text{Division rule}$$

$$2\tfrac{1}{3}h = 7 \qquad \text{C.L.T.}$$

$$h = 3 \qquad \text{Division rule}$$

Joe received 3 half-dollars,
6 quarters,
and 5 dimes.

Check: 3 half-dollars is the same as \$1.50
6 quarters is the same as 1.50
5 dimes is the same as 0.50
Hence the sum is \$3.50

6 is twice 3; 5 is $\frac{5}{3}$ of 3

Hence 3 half-dollars, 6 quarters, and 5 dimes satisfy the conditions of the problem.

Oral Exercises

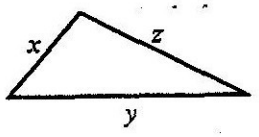
Pick out cue words in the sets of problems on pages 83–86 and state the complete sentence that must be translated as a result of this cue. As an illustration, look at problem 9 on page 85. There the words "sum of the amounts of money" tell us to translate the following sentence: "The number of cents Linda has plus the number of cents Betsy has is equal to 210 cents."

Now solve the problems as directed by your teacher.

Exercises [A-1]

1. The number of boys in the Freshman class is 10 more than the number of girls. The total number of pupils in the class is 152. How many boys and how many girls are in the class?
2. A board whose length is 152 in. is cut into two parts. One part is 10 in. longer than the other. How long is each part?

3. The capacity of a tank is 10 gal. less than the capacity of another. Together they hold 152 gal. How many gallons does each tank hold?
4. Jerry earned \$10 more than Joe. Their total earnings were \$152. How much did each earn?
5. Winken and Blinken were running in a straight line in opposite directions. They passed each other at point A, where Nod was standing. Fifteen seconds after they passed each other they were 152 yards apart. If Winken was then 10 yards farther from Nod than Blinken was, how far was each from Nod?
6. A rectangle is 10 in. longer than it is wide and half its perimeter is 152 in. How long and how wide is the rectangle?
7. Mrs. Shoelady gave her children \$2.40 in quarters, dimes, and nickels. The number of nickels was twice the number of quarters. The number of quarters was twice the number of dimes. How many of each coin did she give out?
8. A rectangle is 5 times as long as it is wide and its width is $27\frac{1}{2}$ inches less than its perimeter. What is its area?
9. Two inches are cut off one side of a square board and three inches are added to an adjacent side. The resulting rectangle has a perimeter of 54 in. What were the dimensions of the original square?
10. Beta is 6 yr. older than Alpha. Three years ago the sum of their ages was 74 yr. How old is each now? How old will each be 10 years from now?
11. In the triangle, y is 2 more than twice x, and z is twice as long as x. The perimeter of the triangle is 22 in. How long is the side z?



12. Gamma earns three times as much per hour as Delta. During one week Gamma works 48 hours and Delta works 14 hours and their total earnings for this week are \$79. How much does each earn per hour?
13. One number is $2\frac{2}{3}$ more than 18 times another. Their difference is $59\frac{1}{3}$. Find the numbers.
14. Six times a number exceeds 200 by 40. What is the number?

Exercises [A–2]

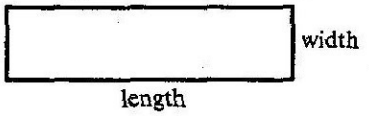
1. The area of one square is $\frac{3}{4}$ that of another, and the sum of their areas is 112 sq. in. Find the area of each.
2. A man had \$10.80 in half-dollars, quarters, dimes, and nickels, with the same number of each kind of coin. How many coins had he?

3. The sum of \$60 is to be divided between two brothers so that the elder gets \$5 more than the younger. How much should each receive?
4. In an election, one candidate received 826 more votes than the other, and the total number of votes was 5486. How many votes were given for each candidate?
5. One flock of sheep contains 32 less than twice the number in a second flock. The two flocks together contain 265 sheep. How many sheep are there in each flock?
6. The capacity of one water tank is $\frac{1}{3}$ of the capacity of another, and together they hold 54 gallons. How much does each tank hold?
7. Divide \$68 among A, B, and C so that A gets three times as much as B, and B gets four times as much as C.
8. A rectangle is three times as long as it is wide, and its perimeter is 115 in. longer than its length. Find its length and width.
9. Linda has a certain number of nickels and Betsy has the same number of pennies. The sum of the amounts of money possessed by the two girls is \$2.10. How much money has each girl?
10. A man left \$8500 for his widow and two daughters and three sons. The widow was to receive \$3250, and each daughter was to receive twice as much as each son. What did each son receive?
11. Tom has twice as many cents as Jim has nickels. Jim has \$2.55 more than Tom. How much has Jim?
12. John is three times as old as Dick, and three years ago the sum of their ages was 22 years. How old is each now?
13. A father earns twice as much per hour as his son. If the father works for 8 hours and the son for 5 hours they earn a total of \$23.10. How much does each earn per hour?
14. Three times a number exceeds 101 by 4. What is the number?

Exercises [A–3]

1. Find three consecutive integers whose sum is 192.
2. A man left \$5000 to be divided among his three daughters and four sons. Each daughter was to receive twice as much as each son. What did each son receive?
3. A sum of \$36.12 was paid out in half-dollars, quarters, dimes, and pennies, there being the same number of each kind of coin. How many coins were needed?

4. A boy has a board 55 in. long and wishes to cut it into two pieces so that one will be 8 in. longer than the other. How long should the pieces be?
5. One number is 2 greater than 3 times another and the sum of the numbers is 46. Find each number.
6. A plank 12 feet long is to be cut into two parts, one of which is three-fifths of the length of the other. Find the length of each of the parts.
7. A rectangle is four times as long as it is wide, and its perimeter is 63 in. longer than its width. Find its area.



8. Three numbers are such that the first is 5 times the second and the second is 3 times the third. The first number is 70 greater than the third. Find the numbers.
9. A table is three times as long as it is wide. If it were 3 feet shorter and 3 feet wider it would be square. Find its length and width.
10. Bob is twice as old as Bill. Three years from now the sum of their ages will be 30 years. How old is each boy now?
11. Tom has the same number of dimes as Jack has cents. Together they have \$2.64. How much money has each boy?
12. There are five large bundles and three small ones. Each large bundle weighs 4 times as much as each small one, and the eight bundles together weigh 34 lb. 8 oz. What is the weight of each bundle?
13. Four times a number exceeds 28 by 35. What is the number?

Exercises [B]

1. Mr. Murgatroyd is employed at an initial salary of \$4000, with annual raises of \$300, while Mr. Wintergreen starts at the same time at \$6000, with annual raises of \$100. After how many years will both men be earning the same salary?
2. The length of a rectangle contains as many feet as the width contains inches, and the perimeter is 19 ft. 6 in. Find the length and width.
3. The length of a rectangular field contains twice as many yards as the width has feet, and the perimeter is 91 yards. Find the width.
4. The level of water in one tank is 1 foot and is increasing at the rate of 3 inches per minute. In a second tank the level is 4 feet and is decreasing at the rate of 3 inches per minute. After how many minutes will the water level in both tanks be the same?

5. There are nine times as many inches in the length of one plank as there are feet in the length of another. The second plank is $4\frac{1}{2}$ inches longer than the first. What is the length of the longer plank?
6. In a group of milk bottles there are four times as many pint bottles as half-pint bottles, and the total amount of milk contained in all the bottles is 63 gal. How many pint bottles are there?
7. The width of a rectangle contains four times as many inches as the length has feet. The perimeter is 4 feet. Find the width.
8. In a set of cubical blocks, some have each edge $1\frac{1}{2}$ in. long, and the others have each edge $2\frac{1}{2}$ in. long. There are twice as many small blocks as there are large ones, and the total volume of all the blocks is 179 cu. in. How many small blocks are there?
9. Seventy exceeds twice a number by an amount that is 5 more than 3 times the number. What is five times the number?

Pages 83-84

1. 81 boys, 71 girls
2. 81 in., 71 in.
3. 81 gal., 71 gal.
4. Jerry, $81; Joe, $71
5. Winken, 81 yd.; Blinken, 71 yd.
6. Length = 81 in.; width = 71 in.
7. 6 quarters, 3 dimes; 12 nickels
8. $31\frac{1}{4}$ sq.in.
9. 13 in. x 13 in.
10. Alpha, 37 yr.; Beta, 43 yr. now;
    Alpha, 47 yr.; Beta, 53 yr. in 10 yr.
11. 8 in.
12. Delta, 50¢ per hr.; Gamma, $1.50 per hr.
13. $3\frac{1}{3}$, $32\frac{2}{3}$
14. 40

18 - Answers to Problems in Text

Pages 84-85

1. 64 sq.in., 48 sq.in.
2. 48 coins
3. Elder, $32.50; younger, $27.50
4. 2330 votes, 3156 votes
5. 99 sheep, 166 sheep
6. $40\frac{1}{2}$ gal., $13\frac{1}{2}$ gal.
7. A, $48; B, $16; C, $4
8. Length, 69 in.; width, 23 in.
9. Linda, $1.75; Betsy, $.35
10. $750
11. $4.25
12. John, 21 yr.; Dick, 7 yr.
13. Father, $2.20 per hr.; son, $1.10 per hr.
14. 35

Pages 85-86

1. 63, 64, 65. 2. $500. 3. 168. 4. $23\frac{1}{2}$ in., $31\frac{1}{2}$ in. 5. 11, 35. 6. $7\frac{1}{2}$ ft., $4\frac{1}{2}$ ft. 7. 196 sq.in. 8. 75, 15, 5. 9. 3 ft., 9 ft. 10. Bill, 8 yr.; Bob, 16 yr. 11. Tom, $2.40; Jack, $.24 12. $1\frac{1}{2}$ lb. or 6 lb. 13. $15\frac{3}{4}$.

Pages 86-87

1. 10 yr.
2. Length, 9 ft.; width, 9 in.
3. $19\frac{1}{2}$ ft.
4. 6 min.
5. 18 in.
6. 448
7. 6 in.
8. 16
9. 65

Pages 88-89

1. 4	4. 5	7. $\frac{1}{2}$	10. $1\frac{1}{2}$	13. $\frac{1}{6}$	16. $\frac{1}{5}$
2. 7	5. 1	8. 3	11. $2\frac{1}{6}$	14. 4	17. 1
3. 1	6. 35	9. $5\frac{1}{2}$	12. $\frac{1}{24}$	15. $\frac{6}{13}$	18. $\frac{1}{4}$

Page 89

1. $6\frac{3}{4}$	4. 0	7. 55	10. 12	13. $30\frac{1}{2}$	16. 19
2. 35	5. $14\frac{1}{2}$	8. $6\frac{1}{2}$	11. 33	14. 38	17. $\frac{4}{13}$
3. 16	6. 49	9. $\frac{1}{12}$	12. $32\frac{1}{2}$	15. $\frac{5}{12}$	18. $\frac{1}{2}$

Page 90 (top)

1. No answer 2. ±3 3. $\pm 2\frac{1}{2}$ 4. $4\frac{2}{3}$

Pages 90-91

1. 8x	6. 6	11. 2x - 8	16. 1
2. 27x	7. 7	12. 6	17. $\frac{1}{3}$
3. x - 2	8. 0	13. 7	18. 1
4. 1 + 2x	9. Yes	14. 8	19. 0
5. 13	10. x - 8	15. 7	20. $7\frac{1}{3}$

Solutions to selected problems

■ **[06-02-20A-P1-7] pps 83-84 [A-1] # 1,4,6,7,9,11**

[1]
Let x = number of girls.
$x + 10$ = number of boys
$x + x + 10 = 152$
$x = 71$
∴ There are 81 boys and 71 girls.

[4]
Let x = amount Joe earned ($).
$x + 10$ = amount Jerry earned ($)
$x + x + 10 = 152$
$x = 71$
∴ Jerry earned $81 and Joe earned $71.

[6]
Let x = width of rectangle (inches).
$x + 10$ = Length of rectangle (inches).
$2x + 2(x + 10)$ = perimeter
$x + x + 10$ = half of perimter
$x + x + 10 = 152$
$x = 33$
∴ The length of the rectangle is 81 inches.The width of the rectangle is 71 inches.

[7]
Let x = the number of dimes.
$2x$ = the number of quarters.
$4x$ = the number of nickels.
$10x$ = the value of the dimes (cents)
$50x$ = the value of the quarters (cents)
$20x$ = the value of the nickels (cents)
$10x + 50x + 20x = 240$
∴ $x = 3$

She had 3 dimes, 6 quarters, and 12 nickels.

[9]
Let x = length of side of original square board (inches)
$x - 2$ = width of rectangular piece (inches)
$x + 3$ = lenght of rectangular piece (inches)
$2(x - 2) + 2(x + 3) = 54$
$x = 13$
∴ The original board was 13 inches by 13 inches

[11]
x = length of one side of the triangle (inches).
$y = 2x + 2$ = length of second side of the triangle (inches).
$z = 2x$ = length of third side of the triangle (inches).
$x + (2x + 2) + (2x) = 22$
$x = 4$
∴ Side z is 8 inches long.

■ **[06-02-20A-P1-7] pps 84-85 [A-2] # 2,3,5,7,9,11,12,13**

[2]
Let x = the number of nickels.
x = the number of dimes.
x = the number of quarters.
x = the number of half-dollars.
$5\,x$ = the value of his nickels (cents).
$10\,x$ = the value of his dimes (cents).
$25\,x$ = the value of his quarters (cents).
$50\,x$ = the value of his half-dollars (cents).
$5\,x + 10\,x + 25\,x + 50\,x = 1080$
$x = 12$

∴ He had 48 coins.

[3]
Let x = amount of money that younger brother recieves ($)
$x + 5$ = amount of money older brother recieves ($).
$x + (x + 5) = 60$
$x = 32.50$
∴ The younger brother recives $27.50, the older gets $32.50

[5]
Let x = the number of sheep in the second flock.
$2\,x - 32$ = the number of sheep in the first flock.
$x + (2\,x - 32) = 265$
$x = 99$
∴ The number of sheep in the first flock is 99. There are 166 sheep in the second flock.

[7]
Let c = the amount of money that C gets ($).
$4\,c$ = the amount of money that B gets ($).
$12\,c$ = the amount of money that A gets ($).
$c + 4\,c + 12\,c = 68$.
$c = 4$
∴ A gets $48, B gets $16, C gets $4.

[8]
Let x = width of rectangle (inches)
$3\,x$ = length of rectangle (inches)
$3\,x + 115$ = perimeter of rectangle (inches).
$2\,x + 2\,(3\,x) = 3\,x + 115$
$x = 23$
∴ The width of the rectangle is 23 inches and its length is 69 inches.

[9]
Let x = the number of Linda's nickels.
x = the number of Betsy's pennies.
$5\,x$ = the value of Linda's nickels (cents)
x = the value of Betsy's pennies (cents)
$x + 5\,x = 210$
$x = 35$
∴ Linda has $1.75 and Betsy has $.35.

[11]
Let x = the number of Jim's nickels.
$2\,x$ = the number of Tom's cents (meaning pennies).

$5x$ = the value of Jim's nickels (cents)
$2x$ = the value of Tom's cents (cents)
$2x + 255 = 5x$
$x = 85$
$\therefore$ Jim has \$4.25. $[85 \times 5 = 425]$

[12]
Let x = Dick's age now (years)
$3x$ = John's age now (years)
$x - 3$ = Dick's age 3 years ago
$3x - 3$ = John's age 3 years ago.
$x - 3 + (3x - 3) = 22$
$x = 7$
$\therefore$ Dick is 7 years old now. John is 21 years old now.

[13]
Let x = amount son earns per hour (\$)
$2x$ = amount father earns per hour (\$)
$5x$ = amount son earns in 5 hours.
$16x$ = amount father earns in 8 hours.
$5x + 16x = 23.10$
$x = 1.10$
$\therefore$ The son earns \$1.10 per hour and the father earns \$2.20 per hour.