$$\frac{dV}{dt} \approx 400\pi(-3) \approx -3770$$

Thus Webster City residents were using water at the rate of $2400 + 3770 = 6170$ cubic feet per hour at 7:00 A.M. ∎

Concepts Review

1. To ask how fast u is changing with respect to time t after 2 hours is to ask the value of ________ at ________.

2. An airplane with a constant speed of 400 miles per hour flew directly over an observer. The distance between the observer and plane grew at an increasing rate, eventually approaching a rate of ________.

3. If dh/dt is decreasing as time t increases, then d^2h/dt^2 is ________.

4. If water is pouring into a spherical tank at a constant rate, then the height of the water grows at a variable and positive rate dh/dt, but d^2h/dt^2 is ________ until h reaches half the height of the tank, after which d^2h/dt^2 becomes ________.

Problem Set 3.9

1. Each edge of a variable cube is increasing at a rate of 3 inches per second. How fast is the volume of the cube increasing when an edge is 12 inches long?

2. Assuming that a soap bubble retains its spherical shape as it expands, how fast is its radius increasing when its radius is 3 inches if air is blown into it at a rate of 3 cubic inches a second?

3. An airplane, flying horizontally at an altitude of 1 mile, passes directly over an observer. If the constant speed of the airplane is 400 miles per hour, how fast is its distance from the observer increasing 45 seconds later? *Hint*: Note that in 45 seconds $(\frac{3}{4}\frac{1}{60} = \frac{1}{80}$ hour), the airplane goes 5 miles.

4. A student is using a straw to drink from a conical paper cup, whose axis is vertical, at a rate of 3 cubic centimeters a second. If the height of the cup is 10 centimeters and the diameter of its opening is 6 centimeters, how fast is the level of the liquid falling when the depth of the liquid is 5 centimeters?

5. An airplane flying west at 300 miles per hour goes over the control tower at noon, and a second airplane at the same altitude, flying north at 400 miles per hour, goes over the tower an hour later. How fast is the distance between the airplanes changing at 2:00 P.M.? *Hint*: See Example 3.

6. A woman on a dock is pulling in a rope fastened to the bow of a small boat. If the woman's hands are 10 feet higher than the point where the rope is attached to the boat and if she is retrieving the rope at a rate of 2 feet per second, how fast is the boat approaching the dock when 25 feet of rope is still out?

7. A 20-foot ladder is leaning against a building. If the bottom of the ladder is sliding along the level pavement directly away from the building at 1 foot per second, how fast is the top of the ladder moving down when the foot of the ladder is 5 feet from the wall?

8. We assume that an oil spill is being cleaned up by deploying bacteria that consume the oil at 4 cubic feet per hour. The oil spill itself is modeled in the form of a very thin cyclinder whose height is the thickness of the oil slick. When the thickness of the slick is 0.001 foot, the cylinder is 500 feet in diameter. If the height is decreasing at 0.0005 foot per hour, at what rate is the area of the slick changing?

9. Sand is pouring from a pipe at the rate of 16 cubic feet per second. If the falling sand forms a conical pile on the ground whose altitude is always $\frac{1}{4}$ the diameter of the base, how fast is the altitude increasing when the pile is 4 feet high? *Hint*: Refer to Figure 8 and use the fact that $V = \frac{1}{3}\pi r^2 h$.

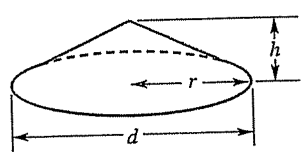


Figure 8

10. A child is flying a kite. If the kite is 90 feet above the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 150 feet of cord is out? (Assume that the cord remains straight from hand to kite, actually an unrealistic assumption.)

11. A swimming pool is 40 feet long, 20 feet wide, 8 feet deep at the deep end, and 3 feet deep at the shallow end; the bottom is rectangular (see Figure 9). If the pool is filled by pumping water into it at the rate of 40 cubic feet per minute, how fast is the water level rising when it is 3 feet deep at the deep end?

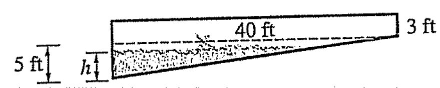


Figure 9

12. A particle P is moving along the graph of $y = \sqrt{x^2 - 4}$, $x \geq 2$, so that the x-coordinate of P is increasing at the rate of 5 units per second. How fast is the y-coordinate of P increasing when $x = 3$?

13. A metal disk expands during heating. If its radius increases at the rate of 0.02 inch per second, how fast is the area of one of its faces increasing when its radius is 8.1 inches?

14. Two ships sail from the same island port, one going north at 24 knots (24 nautical miles per hour) and the other east at 30 knots. The northbound ship departed at 9:00 A.M. and the eastbound ship left at 11:00 A.M. How fast is the distance between them increasing at 2:00 P.M.? *Hint*: Let $t = 0$ at 11:00 A.M.

15. A light in a lighthouse 1 kilometer offshore from a straight shoreline is rotating at 2 revolutions per minute. How fast is the

beam moving along the shoreline when it passes the point $\frac{1}{2}$ kilometer from the point opposite the lighthouse?

C **16.** An aircraft spotter observes a plane flying at a constant altitude of 4000 feet toward a point directly above her head. She notes that when the angle of elevation is $\frac{1}{2}$ radian it is increasing at a rate of $\frac{1}{10}$ radian per second. What is the speed of the airplane?

17. Chris, who is 6 feet tall, is walking away from a street light pole 30 feet high at a rate of 2 feet per second.

(a) How fast is his shadow increasing in length when Chris is 24 feet from the pole? 30 feet?

(b) How fast is the tip of his shadow moving?

(c) To follow the tip of his shadow, at what angular rate must Chris be lifting his eyes when his shadow is 6 feet long?

18. The vertex angle θ opposite the base of an isosceles triangle with equal sides of length 100 centimeters is increasing at $\frac{1}{10}$ radian per minute. How fast is the area of the triangle increasing when the vertex angle measures $\pi/6$ radians? *Hint*: $A = \frac{1}{2}ab\sin\theta$.

≡ **19.** A long, level highway bridge passes over a railroad track that is 100 feet below it and at right angles to it. If an automobile traveling 45 miles per hour (66 feet per second) is directly above a train engine going 60 miles per hour (88 feet per second), how fast will they be separating 10 seconds later?

20. Water is pumped at a uniform rate of 2 liters (1 liter = 1000 cubic centimeters) per minute into a tank shaped like a frustum of a right circular cone. The tank has altitude 80 centimeters and lower and upper radii of 20 and 40 centimeters, respectively (Figure 10). How fast is the water level rising when the depth of the water is 30 centimeters? *Note*: The volume, V, of a frustum of a right circular cone of altitude h and lower and upper radii a and b is $V = \frac{1}{3}\pi h \cdot (a^2 + ab + b^2)$.

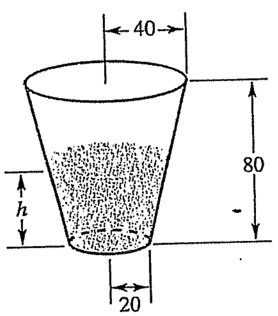


Figure 10

21. Water is leaking out the bottom of a hemispherical tank of radius 8 feet at a rate of 2 cubic feet per hour. The tank was full at a certain time. How fast is the water level changing when its height h is 3 feet? *Note*: The volume of a segment of height h in a hemisphere of radius r is $\pi h^2[r - (h/3)]$. (See Figure 11.)

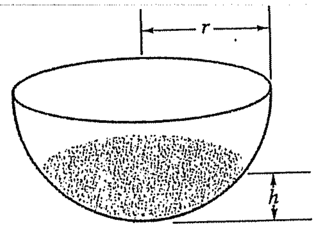


Figure 11

22. The hands on a clock are of length 5 inches (minute hand) and 4 inches (hour hand). How fast is the distance between the tips of the hands changing at 3:00?

23. A right circular cylinder with a piston at one end is filled with gas. Its volume is continually changing because of the movement of the piston. If the temperature of the gas is kept constant, then, by **Boyle's Law**, $PV = k$, where P is the pressure (pounds per square inch), V is the volume (cubic inches), and k is a constant. The pressure was monitored by a recording device over one 10-minute period. The results are shown in Figure 12. Approximately how fast was the volume changing at $t = 6.5$ if its volume was 300 cubic inches at that instant? (See Example 5.)

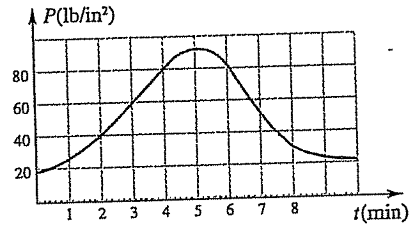


Figure 12

24. Rework Example 5 assuming that the water tank is a sphere of radius 20 feet. (See Problem 21 for the volume of a spherical segment.)

25. Rework Example 5 assuming that the water tank is in the shape of an upper hemisphere. (See Problem 21 for the volume of a spherical segment.)

26. Refer to Example 5. How much water did Webster City use during this 12-hour period from midnight to noon? *Hint*: This is not a differentiation problem.

≡ **27.** An 18-foot ladder leans against a 12-foot vertical wall, its top extending over the wall. The bottom end of the ladder is pulled along the ground away from the wall at 2 feet per second.

(a) Find the vertical velocity of the top end when the ladder makes an angle of 60° with the ground.

(b) Find the vertical acceleration at the same instant.

28. A spherical steel ball rests at the bottom of the tank of Problem 21. Answer the question posed there if the ball has radius

(a) 6 inches and (b) 2 feet.

(Assume that the ball does not affect the flow from the tank.)

29. A snowball melts at a rate proportional to its surface area.

(a) Show that its radius shrinks at a constant rate.

(b) If it melts to $\frac{8}{27}$ its original volume in one hour, how long will it take to melt completely?

30. A steel ball will drop $16t^2$ feet in t seconds. Such a ball is dropped from a height of 64 feet at a horizontal distance 10 feet from a 48-foot street light. How fast is the ball's shadow moving when the ball hits the ground?

31. A girl 5 feet tall walks toward a street light 20 feet high at a rate of 4 feet per second. Her little brother, 3 feet tall, follows at a constant distance of 4 feet directly behind her (Figure 13).

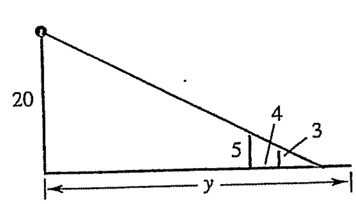


Figure 13

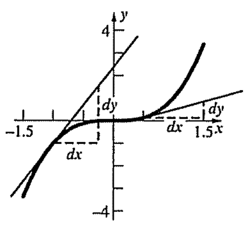
Problem Set 3.9

1. 1296 in.^{3}/s **3.** 392 mi/h **5.** 471 mi/h
7. 0.258 ft/s **9.** 0.0796 ft/s **11.** $\frac{1}{12}$ ft/s
13. 1.018 in.^{2}/s **15.** 15.71 km/min
17. (a) $\frac{1}{2}$ ft/s; **(b)** $\frac{5}{2}$ ft/s **(c)** $\frac{1}{24}$ rad/s
19. 110 ft/s **21.** −0.016 ft/h **23.** 134 in.^{3}/min
25. 4049 ft^3/hr
27. (a) −1.125 ft/s; **(b)** −0.08 ft/s^2
29. (a) Proof omitted; **(b)** 3 hours
31. $\frac{16}{3}$ ft/s when the girl is at least 30 ft from the light pole and $\frac{80}{17}$ ft/s when she is less than 30 ft from the pole.

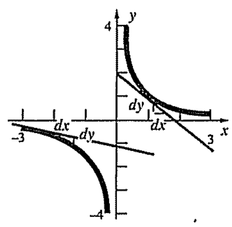
Problem Set 3.10

1. $dy = (2x + 1)dx$ **3.** $dy = -8(2x + 3)^{-5}\,dx$
5. $dy = 3(\sin x + \cos x)^2(\cos x - \sin x)\,dx$
7. $dy = -\frac{3}{2}(14x + 3)(7x^2 + 3x - 1)^{-5/2}\,dx$
9. $ds = \frac{3}{2}(2t + \csc^2 t)\sqrt{t^2 - \cot t + 2}\,dt$

11.



13.



15. (a) $\Delta y = -\frac{1}{3}$ **(b)** $\Delta y = -0.3$
17. (a) $\Delta y = 67$ $dy = 34$
(b) $\Delta y \approx 0.1706$ $dy = 0.17$
19. 5.9917 **21.** 39.27 cm^3 **23.** 893 ft^3
25. 12.6 ft **27.** 4189 ± 63 cm^3; relative error ≈ 0.015
29. 79.097 ± 0.729 cm; relative error ≈ 0.0092
31. $dy = 0.01$; $|\Delta y - dy| \le 0.000003$ **33.** 754 cm^3
35. 9.5% **37. (a)** $L(x) = x$; **(b)** $L(x) = 3x + 4$

Chapter Review 3.11

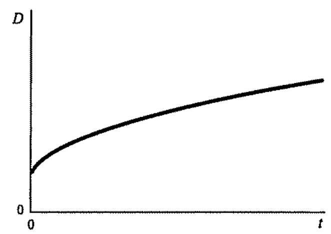
Concepts Test

1. False **3.** True **5.** True **7.** True **9.** True
11. True **13.** False **15.** True **17.** False **19.** True
21. True **23.** True **25.** True **27.** False **29.** True
31. True **33.** True **35.** True **37.** False

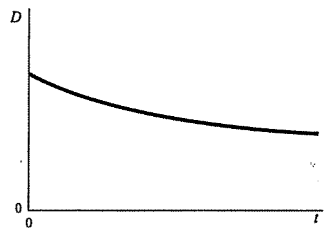
Sample Test Problems

1. (a) $9x^2$; **(b)** $10x^4 + 3$; **(c)** $-\frac{1}{3x^2}$;
(d) $-\frac{6x}{(3x^2+2)^2}$; **(e)** $\frac{3}{2\sqrt{3x}}$; **(f)** $3\cos 3x$;
(g) $\frac{x}{\sqrt{x^2+5}}$; **(h)** $-\pi \sin \pi x$
3. (a) $f(x) = 3x$ at $x = 1$; **(b)** $f(x) = 4x^3$ at $x = 2$;
(c) $f(x) = \sqrt{x^3}$ at $x = 1$; **(d)** $f(x) = \sin x$ at $x = \pi$;
(e) $f(x) = \frac{4}{x}$ at x; **(f)** $f(x) = -\sin 3x$ at x;
(g) $f(x) = \tan x$ at $x = \frac{\pi}{4}$; **(h)** $f(x) = \frac{1}{\sqrt{x}}$ at $x = 5$

5. $15x^4$ **7.** $3z^2 + 8z + 2$ **9.** $\frac{-24t^2 + 60t + 10}{(6t^2 + 2t)^2}$
11. $\frac{-4x^4 + 10x^2 + 2}{(x^3 + x)^2}$ **13.** $-\frac{x}{\sqrt{(x^2+4)^3}}$
15. $-\sin\theta + 6\sin^2\theta\cos\theta - 3\cos^3\theta$ **17.** $2\theta\cos(\theta^2)$
19. $2\pi\sin(\sin(\pi\theta))\cos(\sin(\pi\theta))\cos(\pi\theta)$ **21.** $3\sec^2 3\theta$
23. 672 **25.** $\frac{-\csc^2 x - 2x\cot x\tan x^2}{\sec x^2}$ **27.** $16 - 4\pi$
29. 458.8
31. $F'(r(x) + s(x))(r''(x) + s''(x)) + (r'(x) + s'(x))^2 F''(r(x) + s(x)) + s''(x)$
33. $27z^2\cos(9z^3)$
35.



37.



39. 314 m^3 per meter increase in the radius.
41. 0.167 ft/min
43. (a) $(1,3)$ **(b)** $a(1) = -6, a(3) = 6$; **(c)** $(2,\infty)$
45. (a) $\frac{1-x}{y}$; **(b)** $-\frac{y^2 + 2xy}{x^2 + 2xy}$; **(c)** $\frac{x^2y^3 - x^2}{y^2 - x^3y^2}$;
(d) $\frac{2x - \sin(xy) - xy\cos(xy)}{x^2\cos(xy)}$;
(e) $-\frac{\tan(xy) + xy\sec^2(xy)}{x^2\sec^2(xy)}$
47. 0.0714 **49. (a)** 84; **(b)** 23; **(c)** 20; **(d)** 26
51. 104 mi/h **53. (a)** $\cot\theta|\sin\theta|$; **(b)** $-\tan\theta|\cos\theta|$

Additional Problems 3.12

7. At $x = 0$, $dy = dx$ for both cases. The errors are not always the same sign as dx.
9. (b) $L(x) = \frac{1}{6}x - \frac{13}{30}$; $L(x) = 0$ when $x = \frac{13}{5} = 2.6$, which is less than three, so must be discarded.
(c) $L(x) = 0.25x - 0.85$; $L(x) = 0$ when $x = 3.4$.
(d) $f(x) = 0$ at $x = 3.41$
(e)