

**Problem Set 3.3**

In Problems 1–44, find  $D_x y$  using the rules of this section.

1.  $y = 2x^2$

2.  $y = 3x^3$

3.  $y = \pi x$

4.  $y = \pi x^3$

5.  $y = 2x^{-2}$

6.  $y = -3x^{-4}$

7.  $y = \frac{\pi}{x}$

8.  $y = \frac{\alpha}{x^3}$

9.  $y = \frac{100}{x^5}$

10.  $y = \frac{3\alpha}{4x^5}$

11.  $y = x^2 + 2x$

12.  $y = 3x^4 + x^3$

13.  $y = x^4 + x^3 + x^2 + x + 1$

14.  $y = 3x^4 - 2x^3 - 5x^2 + \pi x + \pi^2$

15.  $y = \pi x^7 - 2x^5 - 5x^{-2}$

16.  $y = x^{12} + 5x^{-2} - \pi x^{-10}$

17.  $y = \frac{3}{x^3} + x^{-4}$

18.  $y = 2x^{-6} + x^{-1}$

19.  $y = \frac{2}{x} - \frac{1}{x^2}$

20.  $y = \frac{3}{x^3} - \frac{1}{x^4}$

21.  $y = \frac{1}{2x} + 2x$

22.  $y = \frac{2}{3x} - \frac{2}{3}$

23.  $y = x(x^2 + 1)$

24.  $y = 3x(x^3 - 1)$

25.  $y = (2x + 1)^2$

26.  $y = (-3x + 2)^2$

27.  $y = (x^2 + 2)(x^3 + 1)$

28.  $y = (x^4 - 1)(x^2 + 1)$

29.  $y = (x^2 + 17)(x^3 - 3x + 1)$

30.  $y = (x^4 + 2x)(x^3 + 2x^2 + 1)$

31.  $y = (5x^2 - 7)(3x^2 - 2x + 1)$

32.  $y = (3x^2 + 2x)(x^4 - 3x + 1)$

33.  $y = \frac{1}{3x^2 + 1}$

34.  $y = \frac{2}{5x^2 - 1}$

35.  $y = \frac{1}{4x^2 - 3x + 9}$

36.  $y = \frac{4}{2x^3 - 3x}$

37.  $y = \frac{x - 1}{x + 1}$

38.  $y = \frac{2x - 1}{x - 1}$

39.  $y = \frac{2x^2 - 1}{3x + 5}$

40.  $y = \frac{5x - 4}{3x^2 + 1}$

41.  $y = \frac{2x^2 - 3x + 1}{2x + 1}$

42.  $y = \frac{5x^2 + 2x - 6}{3x - 1}$

43.  $y = \frac{x^2 - x + 1}{x^2 + 1}$

44.  $y = \frac{x^2 - 2x + 5}{x^2 + 2x - 3}$

45. If  $f(0) = 4$ ,  $f'(0) = -1$ ,  $g(0) = -3$ , and  $g'(0) = 5$ , find  
(a)  $(f \cdot g)'(0)$       (b)  $(f + g)'(0)$       (c)  $(f/g)'(0)$

46. If  $f(3) = 7$ ,  $f'(3) = 2$ ,  $g(3) = 6$ , and  $g'(3) = -10$ , find  
(a)  $(f - g)'(3)$       (b)  $(f \cdot g)'(3)$       (c)  $(g/f)'(3)$

47. Use the Product Rule to show that  $D_x[f(x)]^2 = 2 \cdot f(x) \cdot D_x f(x)$ .

**EXPL** 48. Develop a rule for  $D_x[f(x)g(x)h(x)]$ .

49. Find the equation of the tangent line to  $y = x^2 - 2x + 2$  at the point  $(1, 1)$ .

50. Find the equation of the tangent line to  $y = 1/(x^2 + 4)$  at the point  $(1, 1/5)$ .

51. Find all points on the graph of  $y = x^3 - x^2$  where the tangent line is horizontal.

52. Find all points on the graph of  $y = \frac{1}{3}x^3 + x^2 - x$  where the tangent line has slope 1.

53. Find all points on the graph of  $y = 100/x^5$  where the tangent line is perpendicular to the line  $y = x$ .

54. Prove Theorem F in two ways.

55. The height  $s$  in feet of a ball above the ground at  $t$  seconds is given by  $s = -16t^2 + 40t + 100$ .

(a) What is its instantaneous velocity at  $t = 2$ ?

(b) When is its instantaneous velocity 0?

56. A ball rolls down a long inclined plane so that its distance  $s$  from its starting point after  $t$  seconds is  $s = 4.5t^2 + 2t$  feet. When will its instantaneous velocity be 30 feet per second?

57. There are two tangent lines to the curve  $y = 4x - x^2$  that go through  $(2, 5)$ . Find the equations of both of them. *Hint:* Let  $(x_0, y_0)$  be a point of tangency. Find two conditions that  $(x_0, y_0)$  must satisfy. See Figure 4.

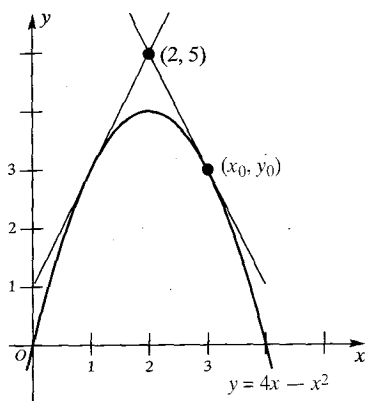


Figure 4

58. A space traveler is moving from left to right along the curve  $y = x^2$ . When she shuts off the engines, she will continue traveling along the tangent line at the point where she is at that time. At what point should she shut off the engines in order to reach the point  $(4, 15)$ ?

59. A fly is crawling from left to right along the top of the curve  $y = 7 - x^2$  (Figure 5). A spider waits at the point  $(4, 0)$ . Find the distance between the two insects when they first see each other.

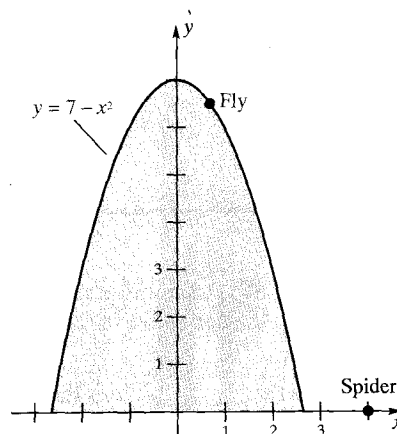


Figure 5

60. Let  $P(a, b)$  be a point on the first quadrant portion of the curve  $y = 1/x$  and let the tangent line at  $P$  intersect the  $x$ -axis at  $A$ . Show that triangle  $AOP$  is isosceles and determine its area.

61. The radius of a spherical watermelon is growing at a constant rate of 2 centimeters per week. The thickness of the rind is always one-tenth of the radius. How fast is the volume of the rind growing at the end of the fifth week? Assume that the radius is initially 0.

62. Redo Problems 29–44 on a computer and compare your answers with those you get by hand.

### Problem Set 3.3

1.  $4x$     3.  $\pi$     5.  $-4x^{-3}$     7.  $-\frac{\pi}{x^2}$     9.  $-\frac{500}{x^6}$
11.  $2x + 2$     13.  $4x^3 + 3x^2 + 2x + 1$
15.  $7\pi x^6 - 10x^4 + 10x^{-3}$     17.  $-\frac{9}{x^4} - 4x^{-5}$
19.  $-\frac{2}{x^2} + \frac{2}{x^3}$     21.  $-\frac{1}{2x^2} + 2$     23.  $3x^2 + 1$
25.  $8x + 4$     27.  $5x^4 + 6x^2 + 2x$
29.  $5x^4 + 42x^2 + 2x - 51$     31.  $60x^3 - 30x^2 - 32x + 14$
33.  $-\frac{6x}{(3x^2 + 1)^2}$     35.  $\frac{-8x + 3}{(4x^2 - 3x + 9)^2}$     37.  $\frac{2}{(x + 1)^2}$
39.  $\frac{6x^2 + 20x + 3}{(3x + 5)^2}$     41.  $\frac{4x^2 + 4x - 5}{(2x + 1)^2}$     43.  $\frac{x^2 - 1}{(x^2 + 1)^2}$
45. (a) 23; (b) 4; (c)  $-\frac{17}{9}$
49.  $y = 1$     51.  $(0, 0)$  and  $(\frac{2}{3}, -\frac{4}{27})$
53.  $(2.817, 0.563)$  and  $(-2.817, -0.563)$
55. (a)  $-24$  ft/s; (b) 1.25 s    57.  $y = 2x + 1, y = -2x + 9$
59.  $3\sqrt{5}$     61. 681  $\text{cm}^3$  per week