

Use mathematical induction to prove the following statements true for every positive integer  $n$ .

$$7. \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$8. \sum_{i=1}^n 2i = n^2 + n$$

$$9. \sum_{i=1}^n (2i-1) = n^2$$

$$10. \sum_{k=1}^n (1+2k) = n^2 + 2n$$

$$11. \sum_{k=1}^n (5+4k) = 2n^2 + 7n$$

$$12. \sum_{k=1}^n [a + (k-1)d] = \frac{n}{2} [2a + (n-1)d]$$

$$13. \sum_{i=1}^n 2^{i-1} = 2^n - 1$$

$$14. \sum_{i=1}^n \left(\frac{1}{2}\right)^i = 1 - \left(\frac{1}{2}\right)^n$$

$$15. \sum_{k=1}^n a \cdot r^{k-1} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$16. \sum_{i=1}^n i^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$17. \sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$$

18.  $n^2 + 5n$  is an even positive integer.

19.  $\frac{1}{2}(n+2)(n+1)$  is a positive integer.

20.  $\frac{1}{3}(n^3 - n + 3)$  is a positive integer.

21.  $\frac{1}{5}(9^n - 4^n)$  is a positive integer.

22. If  $0 < a < b$ , then  $a^n < b^n$ .

23. The square of every odd integer greater than 1 can be written in the form  $4p + 1$ , where  $p$  is a positive integer.

24.  $i^n \in \{i, -1, -i, 1\}$  for any integer  $n$ . (*Hint*: Use mathematical induction to prove the statement for any natural number  $n$  and go on from there.)