1.1 Radian measure

The degree is one of several arbitrary measures of angle. It is convenient for applied work. On the other hand, there is a natural measure of angle called the radian that is more convenient than the degree for mathematics and theoretical work.

Definition 1.1. Given a circle of radius r, the central angle of **one radian** is subtended by an arc of length r.

Several useful facts follow almost immediately.

Let s be an arc of a circle of radius r, and let θ the central angle subtended by arc s.

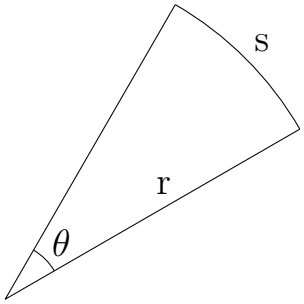

Figure 1.1: Sector of circle radius r, arc length s, central angle θ.

Then

$$\theta = \frac{s}{r} \qquad \text{or equivalently} \qquad s = r\theta. \tag{1.1}$$

Let A_s be the area of the sector of figure (1.1). Since the area of a sector of a circle should compare to the circle's area as the length of the arc compares to the circumference of the circle,

$$\frac{A_s}{\pi r^2} = \frac{s}{2\pi r}$$

which implies that

$$A_s = \frac{1}{2} rs \tag{1.2}$$

and by substitution

$$A_s = \frac{1}{2} r^2\theta. \tag{1.3}$$

Equations (1.2) and (1.3) are often useful.