# [12-10-24-T11] REV

*Notes for exam on basic trigonometry*

Topics include those of the textbook Basic Analysis Chapter 2 excluding pages 57-58 the sum of trigonometric functions.

- Calculators will not be allowed.
- Notes will not be allowed.
- The sheet of basic identities will not be allowed.

## ■ Circle

- Convert radians to degrees and degrees to radians.

- Know and be able to apply the relationship of arc length $s$ and central angle $\theta$ on a circle radius $r$; that is, $s = r\,\theta$.

## ■ Unit circle

- Know the definitions of all six trigonometric functions. Especially that the $\cos x$, $\sin x$ are defined for all $x \in \mathbb{R}$ as the first and second coordinates of the point P at the end of an arc length $x$ on the unit circle.

- Know and use all the identities that are based on the symmetry and periodicity of the unit circle. Side one of the sheet of identities.

- Know by heart the values of the sine and cosine functions at arguments 0, $\frac{\pi}{2}$, $\pi$, $\frac{3\pi}{2}$, $2\pi$, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$.

- Use the identities based on the symmetry and periodicity of the unit circle to determine the values of the trigonometric functions for selected arguments other than 0, $\frac{\pi}{2}$, $\pi$, $\frac{3\pi}{2}$, $2\pi$, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$. For example, $\sin\frac{5\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$.

- State all the arguments of a trigonometric function, given the value of the function. For example, given that $\sin x = \frac{-\sqrt{3}}{2}$, you should conclude that $x = \frac{4\pi}{3} + 2\,n\,\pi,\ n \in \mathbb{Z}$ or $x = \frac{5\pi}{3} + 2\,n\,\pi,\ n \in \mathbb{Z}$.

- State all the arguments within the interval $[0, 2\pi]$ of a trigonometric function for which the value of the function is greater than, less than, no greater than, or no less than a given number. For example, given that $\sin x < \frac{-\sqrt{3}}{2}$, you should conclude that $x \in \left(\frac{4\pi}{3}, \frac{5\pi}{3}\right)$.

## ■ The function $f(x) = A\sin k(x - \beta) + B$

- Rewrite a function in the canonical form $f(x) = A\sin k(x - \beta) + B$.

- Know how the various constants affect the behavior of the function $f(x) = A\sin k(x - \beta) + B$.

- Use the relationship of frequency $k$ and period $T$. In general $k = \frac{c}{T}$ where $c$ is the fundamental period of a periodic function. In the case of the sine and cosine functions, $k = \frac{2\pi}{T}$.

- Graph the function $f(x) = A \sin k(x - \beta) + B$ for various values of the constants. Label the values of $x$ for which $f(x)$ is zero, a minimum, or a maximum. The maximum and minimum values of the function must also be shown. Include at least complete period of the function.

- Graph the sine, cosine, tangent, secant, cosecant, and cotangent functions. Vertical asymptotes and local minimum and maximum values must be shown and labeled.

- Given the graph, write the function in the form $f(x) = A \sin k(x - \beta) + B$.

## ■ Identities based on addition theorem.

- Give geometric proof of an addition theorem. The proof of $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \cos\beta$ in the book is good. That is also the proof we gave in class.

- Prove the remaining three addition theorems.

- Prove any of the identities on the second side of the identity sheet by using only the four addition theorems.

- Know by heart and use the identities on the second side of the identity sheet to find the values of the trigonometric functions at selected arguments. For example,
$\sin\frac{\pi}{12} = \sin(\frac{\pi}{3} - \frac{\pi}{4}) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$.

## ■ Proof

- Prove identities that are not on the sheet of identities. You may freely use any of the identities on the two sided sheet of basic identities. But, as noted, you will not have that sheet available during the exam.