[10-11-19-T]

Combinations

[1] Without using a calculator, compute:

(a) $\binom{9}{3}$

(b) $\binom{10}{2}$

[2] A class contains 9 boys and 3 girls.

(a) In how many ways can the teacher choose a committee of 4?

(b) How many of them will contain at least one girl?

(c) How many of them will contain exactly one girl?

[2] A woman has 11 friends.

(a) In how many ways can she invite 5 of them to dinner?

(b) In how many ways if two of the friends are married and will not attend separately?

(c) In how many ways if two of them are not on speaking terms and will not attend together?

[3] There are 10 points A, B, C, ... in a plane, no three of them on the same line.

(a) How many lines are determined by the points?

(b) How many of these lines do not pass through A or B?

(c) How many triangles are determined by the points?

(d) How many of these triangles contain point A?

(e) How many of these triangles contain side AB?

[4] A student is to answer 10 out of 13 questions on an exam.

(a) How many choices has he?

(b) How many if he must answer the first two questions?

(c) How many if he must answer the first or second question but not both?

(d) How many if he must answer exactly 3 of the first 5 questions?

(e) How many if he must answer at least 3 of the first 5 questions?

[5] A man is dealt a five card hand from an ordinary deck of playing cards.

(a) In how many ways can he be dealt 5 cards of the same suit having sequential rank? For example, 3H,4H,5H,6G,7H?

(b) Four of a kind?

(c) A sequence of 5 cards? For example, 2,3,4,5,6 of any suits.

(d) A pair of Aces?

(e) Exactly two of a kind (a pair)?

Answers

[1] Without using a calculator, compute:

(a) 84

(b) 45

[2] A class contains 9 boys and 3 girls.

(a) $\binom{12}{4} = 495$

(b) $\binom{12}{4} - \binom{9}{4} = 396$

(c) $3\binom{9}{3} = 252$

[2] A woman has 11 friends.

(a) $\binom{11}{5} = 462$

(b) $\binom{9}{3} + \binom{9}{5} = 210$

(c) $\binom{9}{5} + 2 \cdot \binom{9}{4} = 378$

[3] There are 10 points A, B, C, ... in a plane, no three of them on the same line.

(a) $\binom{10}{2} = 45$

(b) $\binom{8}{2} = 28$

(c) $\binom{10}{3} = 120$

(d) $\binom{9}{2} = 36$

(e) 8

[4] A student is to answer 10 out of 13 questions on an exam.

(a) $\binom{13}{10} = \binom{13}{3} = 286$

(b) $\binom{11}{8} = \binom{11}{3} = 165$

(c) $2 \cdot \binom{11}{9} = 2 \cdot \binom{11}{2} = 110$

(d) $\binom{5}{3}\binom{8}{7} = 80$

(e) $\binom{5}{3}\binom{8}{7} + \binom{5}{4}\binom{8}{6} + \binom{5}{5}\binom{8}{5}$

[5] A man is dealt a five card hand from an ordinary deck of playing cards.

(a) $4 \cdot 10 = 40$

(b) $13 \cdot 48 = 624$

(c) $4^5 - 40 = 10200.$

(d) $\binom{4}{2}\binom{12}{3} \cdot 4^3 = 84480$

(e) $13 \cdot \binom{4}{2}\binom{12}{3} \cdot 4^3 = 1098240$