MATH 143 - PROBLEM SET A

1. a) How many 4-digit numbers can be formed from the digits 1, 2, 3, 4, using each digit only once?
   b) How many of these numbers are less than 3,000?

2. We must arrange nine different books on a shelf. Three books are green, two are red, and four are blue.
   a) How many ways can this be done if there are no restrictions?
   b) If books of the same color must be grouped together?
   c) If books of the same color are together and red books must be to the left of blue books?
   d) If, in addition to the restrictions in b) and c), red and blue books must not be next to each other?

3. Fifteen students register for three sections of Math 143.
   a) How many ways can the students distribute themselves among the sections?
   b) How many ways given the same number of students in each section?

4. A group of ten people is to be divided into three committees of three, three, and six members, respectively. The chairman of the group is to serve on all three committees and is the only member of the group who serves on more than one committee. How many ways can the committee assignments be made?

5. A student must choose two electives from six possible courses. In how many ways:
   a) can he make his choice?
   b) if two of the courses meet at the same time?
   c) if two of the courses meet at 10:00 o'clock, two at 11:00 o'clock, and there are no other conflicts among the courses?

6. John, Joan, Jim, and Judy have formed a band consisting of 4 instruments.
   a) If each member can play all 4 instruments, how many different arrangements are possible?
   b) What if John and Joan can play all 4 instruments, but Jim and Judy can each play only piano and drums?

7. How many anagrams are there of the words:
   a) FLUKE b) PROPOSE c) MISSISSIPPI d) VISITING ?

8. a) How many ways can 5 people be lined up in a row for a group picture?
   b) How many ways if there are to be 3 people in the front row and 2 in the back row?

9. Five different awards are to be presented to selected students from a class of 30. How many different outcomes are possible if:
   a) a student can receive any number of awards?
   b) each student can receive, at most, one award?

10. How many signals can a ship show if it has 7 different flags and a signal consists of 5 flags hoisted vertically on a rope?

11. How many license plates can be made if they are to contain 5 symbols, the first 2 being letters and the last 3 being digits?

12. a) How many ways can a 10-question multiple choice test be answered if each question has 3 possible responses: a, b, and c?
   b) If no 2 consecutive answers are the same?

13. How many 4-digit odd numbers can you form without repeating any digit and without using zero for the first digit?

14. How many ways can a man divide 7 different gifts among his 3 children if the eldest is to receive 3 gifts and the others 2 each?

15. A chess tournament has 10 competitors of which 4 are Russian, 3 are from the United States, 2 are from Britain, and 1 is from Brazil. If the tournament result just lists the nationalities of the players in the order in which they placed, how many outcomes are possible?

16. a) From a group of 5 men and 7 women, how many different committees, consisting of 2 men and 3 women, can be formed?
    b) What if 2 of the women are feuding and refuse to serve on the committee together?

[17.] A student is to answer 7 out of 10 questions on an examination.
    a) How many choices does he have?
    b) How many if he must answer at least 3 of the first 5 questions?

18. How many different signals, each consisting of 9 flags hung in a line, can be made from a set of 4 white flags, 3 red flags, and 2 blue flags, if all flags of the same color are identical?

19. Using the letters of the word EQUATION (without repetition), how many 4-letter code words can be formed:
    a) starting with T and ending with N?
    b) starting and ending with a consonant?
    c) with vowels only?
    ? → d) with 3 consonants?
    e) with all the vowels (if any are used) in the right-hand portion of the word?

20. An ice cream parlor has 10 different flavors. How many different double dip cones can be made if:
    a) the 2 dips must be of different flavors, but the order of putting them on the come doesn't matter?
    b) the 2 dips must be different and order does matter?
    c) the 2 dips need not be different but order matters?
    d) the 2 dips need not be different but order doesn't matter?

21. How many committees of 2 Republicans, 2 Democrats, and 3 Independents can be chosen from a group of 5 Republicans, 6 Democrats, and 4 Independents?

22. a) How many ways can 8 new teachers be assigned to 4 schools?
    b) How many ways if each school must receive 2 teachers?

23. A certain town of population 100,000 has 3 newspapers: I, II, and III. The proportions of townspeople that read these papers are:

√I: 10%	√I and II: 8%	√I and II and III: 1%
√II: 30%	√I and III: 2%	
III: 5%	√II and III: 4%	

    a) How many people read only one paper?
    b) How many read at least 2 papers?
    c) If I and III are morning papers and II is an evening paper, how many people read at least one morning paper plus the evening paper?
    (d) How many people read only one morning paper and the evening paper?

why am I wrong @ 10,000?

24. A committee of 4 is to be chosen from a club which boasts a membership of 10 men and 12 women. How many ways can the committee be formed if it must contain at least 2 women?

25. There are 9 different books on a shelf: 4 are red and 5 are green. How many ways can the books be arranged on the shelf if:
    a) there are no restrictions?
    b) books of the same color must be together?
    c) the red books must be together but the green books need not be?
    d) the colors must alternate?

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26. From among a group of 6 men and 9 women, how many 3-member committees contain only men or only women?

27. Suppose that 3 freshmen, 5 sophomores, and 4 juniors have been nominated to receive scholarships of $500, $250, and $100. How many ways can the scholarships be awarded if:
    a) anyone may receive any scholarship?
    b) the $500 scholarship goes to a freshman, the $250 scholarship to a sophomore, and the $100 scholarship to a junior?
    c) at least 2 scholarships are to be awarded to juniors?
    d) one scholarship is to be awarded to someone from each class?

28. How many ways can 3 married couples be seated in a row of 6 chairs if:
    a) there are no restrictions?
    b) men must occupy the first and last chairs?
    c) the men must sit to the left of the women?
    d) couples must sit together?

29. How many 5-card poker hands:
    a) have exactly 2 hearts and 2 diamonds?
    b) have 2 pairs of different kinds, (e.g., 2 aces and 2 fives)?
    c) are straights (e.g., 7, 8, 9, 10, jack)? An ace may count as either the highest or the lowest card, (i.e., as 1 or 14).
    d) contain exactly 3 spades?
    e) contain exactly 3 cards of some suit?
    f) contain cards of all 4 suits?

30. How many ways can 12 different books be distributed among 4 children if:
    a) each child gets 3 books?
    b) the oldest children get 4 books each while the 2 youngest get 2 books each?

31. How many 4-letter code words made from the letters H I S T O R Y:
    a) are there altogether?
    b) contain only consonants (H, S, T, R, Y)?
    c) begin and end with a consonant?
    d) begin with a vowel?
    e) contain the letter Y?
    f) begin with T and end in a vowel?
    g) begin with T and also contain S?
    h) contain both vowels?

32. A woman has 11 close friends. How many ways can she invite 5 of them to dinner:
    a) altogether?
    b) if 2 of the friends are married and will not attend separately?
    c) if 2 of them are not on speaking terms and will not attend together?

33. Ralph is to answer 10 out of 13 questions on an exam. How many ways can he choose the 10 questions:
    a) altogether?
    b) if he must answer the first 2 questions?
    c) if he must answer the first or the second question, but not both?
    d) if he must answer exactly 3 of the first 5 questions?
    e) if he must answer at least 3 of the first 5 questions?

34. a) How many ways can 3 boys and 3 girls sit in a row?
    b) If the boys must sit together and the girls must sit together?
    c) If only the boys must sit together?
    d) If no 2 people of the same sex can sit together?

35. A child has 12 blocks, of which 6 are black, 4 are red, 1 is white, and 1 is blue. If blocks of the same color are identical, how many ways can the 12 blocks be arranged in a line?

36. A president, treasurer, and secretary, all different, are to be chosen from a club consisting of 10 people. How many choices of officers are possible if:
    a) there are no restrictions?
    b) A and B will not serve together?
    c) C and D will serve together or not at all?
    d) E must be an officer?
    f) F will serve only if she is president?

37. Suppose 45% of all students like wine, 60% like beer, and 55% like tea. Suppose 35% like any given pair of these beverages and 25% like all 3 beverages. What percent of students like:
    a) only beer? b) only 2 beverages? c) tea but not wine?
    d) only 1 beverage? e) beer, but not tea? f) wine and tea, but not beer?
    g) beer or tea? h) tea or wine?

38. A committee is to be chosen from a group of 7 women and 4 men. How many ways are there to form the committee if:
    a) the committee has 3 women and 2 men?
    b) the committee can be any size (except empty), but must have equal numbers of men and women?
    c) the committee has 4 people and 1 of them must be Mr. Smith?
    d) the committee has 2 men and 2 women and Mr. and Mrs. Smith cannot both be on the committee?

39. A license plate must contain a sequence of 2 letters followed by 4 digits or 3 letters followed by 3 digits. How many different license plates can be made?

40. Using the letters of the word FACTOR (without repetition), how many 4-letter code words can be formed:
    a) starting with R?
    b) with vowels in the 2 middle positions?
    c) with only consonants?
    d) with vowels and consonants alternating (in either order)?
    e) with all the vowels (if any are used) in the left-hand portion of the word (i.e., a vowel cannot be preceded by a consonant)?

41. How many different signals, each consisting of 8 flags hung in a vertical line, can be formed from 4 red flags, 2 blue flags, and 2 green flags, if flags of the same color are identical?

42. Consider all positive integers with 3 different digits (0 cannot be the first digit). How many are:
    a) there altogether?
    b) greater than 700?
    c) odd?
    d) even?
    e) divisible by 5?

43. If 4 Americans, 3 Frenchmen, and 3 Englishmen are to be seated in a row, how many seating arrangements are possible when people of the same nationality must sit together?

44. A woman has 8 friends, of whom she will invite 5 to a tea party.
    a) How many choices of guests does she have if 2 of the friends are feuding and will not attend together?
    b) How many if 2 of her friends will only attend together?

45. How many ways can 8 people be seated in a row if:
    a) there are no restrictions?
    b) A and B must sit next to each other?
    c) there are 4 men and 4 women and the sexes must alternate?
    d) there are 5 men and they must sit together?
    e) there are 4 couples and each couple must sit together?

46. Grades of A, B, C, D, and F are assigned to a class of 5 students.
    a) How many ways can this be done if no two students receive the same grade?
    b) How many ways if Clio must receive a higher grade than Ralph and no 2 students receive the same grade?
    c) How many ways if only grades of A and F are given?

47. How many ways can 3 different novels, 2 different math books, and 2 chemistry book be arranged on a shelf if:
    a) there are no restrictions?
    b) the math books must be together and the novels must be together?
    c) the novels must be together but the other books can be arranged in any order?

48. A college professor plans to teach the same course for the next 35 years. In order not to become bored with his jokes, he decides to tell exactly 3 jokes every year and never to tell the same 3 jokes in two different years.
    a) How many jokes must he learn in order to accomplish this?
    b) How many if he resolves never to tell the same joke twice?

49. In the state of Bliss all license plates are made up of 6 different digits selected from 1 through 9, but not including 0. Until recently this was also true in the state of Ignorance. But now the governor of Ignorance has decreed that 3 of the 6 characters on every license plate must be his initials: GCF (in any order and in any position on the plate, (e.g., C4G93F). The other 3 characters must still be 3 different digits (1 through 9, but not 0). Which state can generate the largest number of license plates?

1. 24; 12
2. 362,880; 1728; 864; 288
3. ~~315~~; 756,756
4. 756
5. 15; 14; 13
6. 24; 4
7. 120; 1260; 34,650; 6720
8. 120; 120
9. 24,300,000; 17,100,720
10. 2520
11. 676,000
12. 59,049; 1536
13. 2240
14. 210
15. 12,600
16. 350; 300
17. 120; 110
18. 1260
19. 30; 180; 120; 120; 450
20. 45; 90; 100; 55
21. 600
22. 65,536; 2520
23. 20,000; 12,000; 11,000; 18,000
24. 5665
25. 362,880; 5760; 17,280; 2880
26. 104
27. 1320; 60; 312; 360
28. 720; 144; 36; 48
29. 1584; 123,552; 10,240; 211,926; 847,704; 685,464
30. 369,600; 207,900
31. 840; 120; 400; 240; 480; 40; 60; 240
32. 462; 210; 378
33. 286; 165; 110; 80; 276
34. 720; 72; 144; 72
35. 27,720
36. 720; 672; 384; 216; 576
37. 15, 30, 20; 25; 25; 10; 80; 65
38. 210; 329; 120; 108
39. 24,336,000
40. 60; 24; 24; 48; 96
41. 420
42. 648; 216; 320; 328; 136
43. 5184
44. 36; 26
45. 40,320; 10,080; 1152; 2880; 384
46. 120; 60; 32
47. 120; 72; 144
48. 7; 105
49. 60,480 each