3 Find the sum of these arithmetic series:

a $\sum_{r=1}^{10}(2r+5)$ **b** $\sum_{r=1}^{15}(r-50)$ **c** $\sum_{r=1}^{20}\left(\frac{r+3}{2}\right)$

Hint: List the first 3 terms and the last term.

4 Find the sum of these geometric series:

a $\sum_{r=1}^{10} 3\times 2^{r-1}$ **b** $\sum_{r=1}^{12}(\frac{1}{2})^{r-2}$ **c** $\sum_{r=1}^{25} 6\times(-2)^r$

Hint: List the first 3 terms and the last term.

5 Find the sum of: **a** $\sum_{k=1}^{5} k(k+1)(k+2)$ **b** $\sum_{k=6}^{12} 100\times(1.2)^{k-3}$

6 Find n given that: **a** $\sum_{r=1}^{n}(2r+3)=1517$ **b** $\sum_{r=1}^{n} 2\times 3^{r-1}=177\,146$

G MISCELLANEOUS PROBLEMS

EXERCISE 2G

1 Henk starts a new job selling TV sets. He hopes to sell 11 sets in the first week, 14 in the next, 17 in the next, etc., in arithmetic sequence. In what week will Henk hope to sell his 2000th TV set?

2 A computer is bought for \$2795 and depreciates at a rate of 2% per month. After how many months will its value reduce to \$500?

3 A geometric series has a second term of 6 and the sum of its first three terms is -14. Find its fourth term.

4 When a ball falls vertically off a table it rebounds 75% of its height after each bounce. If it travels a total distance of 490 cm, how high was the table top above the floor?

5 An arithmetic and a geometric sequence both have a first term of 1 and their second terms are equal. The 14th term of the arithmetic sequence is three times the third term of the geometric sequence. Find the twentieth term of each sequence.

NO **6** Find x if $\sum_{r=1}^{\infty}\left(\frac{3x}{2}\right)^{r-1}=4$.

7 The sum of the first n terms of an arithmetic sequence is $\frac{n(3n+11)}{2}$.

a Find its first two terms. **b** Find the twentieth term of the sequence.

8 **Mortgage repayments:**

\$8000 is borrowed over a 2-year period at a rate of 12% p.a. Quarterly repayments are made and the interest is adjusted each quarter, which means that the amount repaid in the period is deducted and the interest is charged on the new amount owed.

There are $2\times 4=8$ repayments and the interest per quarter is $\frac{12\%}{4}=3\%$.

At the end of the first quarter the amount owed, A_1, is given by $\$8000\times 1.03-R$, where R is the amount of each repayment.

At the end of the second quarter the amount owed, A_2, is given by

$$\begin{aligned} A_2 &= A_1 \times 1.03 - R \\ &= (\$8000 \times 1.03 - R) \times 1.03 - R \\ &= \$8000 \times (1.03)^2 - 1.03R - R \end{aligned}$$

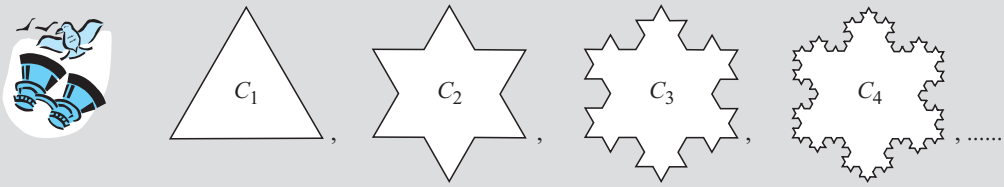
a Find a similar expression for the amount owed at the end of the third quarter, A_3.

b Write down an expression for the amount owed at the end of the 8th quarter, A_8, and hence deduce the value of R. [**Hint:** What value do we want A_8 to have?]

c If the amount borrowed is $\$P$ at adjusted interest conditions, the interest rate is $r\%$ per repayment interval and there are m repayments, show that the amount of each repayment is

$$R = \frac{P(1 + \frac{r}{100})^m \times \frac{r}{100}}{(1 + \frac{r}{100})^m - 1}.$$

INVESTIGATION VON KOCH'S SNOWFLAKE CURVE



To draw **Von Koch's Snowflake curve** we

- start with an equilateral triangle, C_1
- then divide each side into 3 equal parts
- then on each middle part draw an equilateral triangle
- then delete the side of the smaller triangle which lies on C_1.

The resulting curve is C_2, and C_3, C_4, C_5, ... are found by 'pushing out' equilateral triangles on each edge of the previous curve as we did with C_1 to get C_2.

We get a sequence of special curves C_1, C_2, C_3, C_4, ... and Von Koch's curve is the limiting case, i.e., when n is infinitely large for this sequence.

Your task is to investigate the perimeter and area of Von Koch's curve.

What to do:

1 Suppose C_1 has a perimeter of 3 units. Find the perimeter of C_2, C_3, C_4 and C_5.

(**Hint:** becomes i.e., 3 parts become 4 parts.)

Remembering that Von Koch's curve is C_n, where n is infinitely large, find the perimeter of Von Koch's curve.

2 Suppose the area of C_1 is 1 unit^2. Explain why the areas of C_2, C_3, C_4 and C_5 are

$A_2 = 1 + \frac{1}{3}$ units^2 $\qquad$ $A_3 = 1 + \frac{1}{3}[1 + \frac{4}{9}]$ units^2

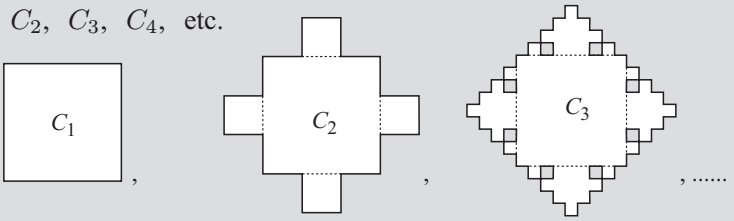
$A_4 = 1 + \frac{1}{3}[1 + \frac{4}{9} + (\frac{4}{9})^2]$ units^2 $\qquad$ $A_5 = 1 + \frac{1}{3}[1 + \frac{4}{9} + (\frac{4}{9})^2 + (\frac{4}{9})^3]$ units^2.

Use your calculator to find A_n where $n = 1, 2, 3, 4, 5, 6, 7$, etc., giving answers which are as accurate as your calculator permits.

What do you think will be the area within Von Koch's snowflake curve?

3 Similarly, investigate the sequence of curves obtained by *pushing out* squares on successive curves from the middle third of each side,

i.e., the curves C_1, C_2, C_3, C_4, etc.



Region contains 8 holes.

REVIEW SET 2A

1 List the first four members of the following sequences defined by:

a $u_n = 3^{n-2}$ **b** $u_n = \dfrac{3n+2}{n+3}$ **c** $u_n = 2^n - (-3)^n$

2 A sequence is defined by $u_n = 68 - 5n$.

a Prove that the sequence is arithmetic. **b** Find u_1 and d. **c** Find the 37th term.
d What is the first term of the sequence less than -200?

3 **a** Show that the sequence 3, 12, 48, 192, ... is geometric.
b Find u_n and hence find u_9.

4 Find k if $3k$, $k-2$ and $k+7$ are consecutive terms of an arithmetic sequence.

5 Find the general term of an arithmetic sequence given that $u_7 = 31$ and $u_{15} = -17$. Hence, find the value of u_{34}.

6 A sequence is defined by $u_n = 6(\frac{1}{2})^{n-1}$.

a Prove that the sequence is geometric. **b** Find u_1 and r.
c Find the 16th term to 3 significant figures.

7 Show that 28, 23, 18, 13, ... is arithmetic and hence find u_n and the sum S_n of the first n terms in simplest form.

8 Find k given that 4, k and $k^2 - 1$ are consecutive geometric terms.

9 Determine the general term of a geometric sequence given that its sixth term is $\frac{16}{3}$ and its tenth term is $\frac{256}{3}$.

REVIEW SET 2B

1 **a** Determine the number of terms in the sequence $24,\ 23\frac{1}{4},\ 22\frac{1}{2},\ ....,\ -36$.
b Find the value of u_{35} for the sequence in **a**.
c Find the sum of the terms of the sequence in **a**.

2 Insert six numbers between 23 and 9 so that all eight numbers are in arithmetic sequence.

3 Find the formula for u_n, the general term of:

a 86, 83, 80, 77, ... **b** $\frac{3}{4}$, 1, $\frac{7}{6}$, $\frac{9}{7}$, ... **c** 100, 90, 81, 72.9, ...

[**Note:** One of these sequences is neither arithmetic nor geometric.]

4 Write down the expansion of: **a** $\sum_{r=1}^{7} r^2$ **b** $\sum_{r=1}^{8} \frac{r+3}{r+2}$

5 Write in the form $\sum_{r=1}^{n}(.....)$:

a $4 + 11 + 18 + 25 + ...$ for n terms **b** $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$ for n terms.

6 Find the sum of:

a $3 + 9 + 15 + 21 + ...$ to 23 terms **b** $24 + 12 + 6 + 3 + ...$ to 12 terms.

7 Find the sum of **a** $\sum_{r=1}^{8}\left(\frac{31-3r}{2}\right)$ **b** $\sum_{r=1}^{15} 50(0.8)^{r-1}$

8 Find the first term of the sequence 5, 10, 20, 40, ... which exceeds 10 000.

9 What will an investment of 6000 Euro at 7% p.a. compound interest amount to after 5 years if the interest is compounded:

a annually **b** quarterly **c** monthly?

REVIEW SET 2C

1 A geometric sequence has $u_6 = 24$ and $u_{11} = 768$. Determine the general term of the sequence and hence find:

a u_{17} **b** the sum of the first 15 terms.

2 How many terms of the series $11 + 16 + 21 + 26 + ...$ are needed to exceed a sum of 450?

3 Find the first term of the sequence 24, 8, $\frac{8}{3}$, $\frac{8}{9}$, ... which is less than 0.001.

4 **a** Determine the number of terms in the sequence 128, 64, 32, 16, ..., $\frac{1}{512}$.

b Find the sum of these terms.

5 \$12 500 is invested in an account which pays 8.25% p.a. compounded. Find the value of the investment after 5 years if the interest is compounded:

a half-yearly **b** monthly.

6 How much should be invested at a fixed rate of 9% p.a. compounded interest if you wish it to amount to \$20 000 after 4 years with interest paid monthly?

7 In 1998 there were 3000 koalas on Koala Island. Since then, the population of koalas on the island has increased by 5% each year.

a How many koalas were on the island in 2001?

b In what year will the population first exceed 5000?

NO **8** A ball bounces from a height of 2 metres and returns to 80% of its previous height on each bounce. Find the total distance travelled by the ball until it stops bouncing.

NO **9** **a** Under what conditions will the series $\sum_{r=1}^{\infty} 50(2x-1)^{r-1}$ converge? Explain!

b Find $\sum_{r=1}^{\infty} 50(2x-1)^{r-1}$ if $x = 0.3$.

e Range of g $\{y: y \geqslant -2\}$, Domain of g^{-1} $\{x: x \geqslant -2\}$
Range of g^{-1} $\{y: y \leqslant -3\}$

8 **a** $h^{-1}(x) = 4 + \sqrt{x-3}$

9 $(f^{-1} \circ h^{-1})(x) = x - 2$ and $(h \circ f)^{-1}(x) = x - 2$

EXERCISE 2A

1 **a** 4, 13, 22, 31, ... **b** 45, 39, 33, 27, ...
c 2, 6, 18, 54, ... **d** 96, 48, 24, 12, ...

2 **a** Starts at 8 and each term is 8 more than the previous term. Next two terms 40, 48.

b Starts at 2, each term is 3 more than the previous term; 14, 17.

c Starts at 36, each term is 5 less than the previous term; 16, 11.

d Starts at 96, each term is 7 less than the previous term; 68, 61.

e Starts at 1, each term is 4 times the previous term; 256, 1024.

f Starts at 2, each term is 3 times the previous term; 162, 486.

g Starts at 480, each term is half the previous term; 30, 15.

h Starts at 243, each term is $\frac{1}{3}$ of the previous term; 3, 1.

i Starts at 50 000, each term is $\frac{1}{5}$ of the previous term; 80, 16.

3 **a** Each term is the square of the number of the term; 25, 36, 49.

b Each term is the cube of the number of the term; 125, 216, 343.

c Each term is $n \times (n+1)$ where n is the number of the term; 30, 42, 56.

EXERCISE 2B

1 **a** 2, 4, 6, 8, 10 **b** 4, 6, 8, 10, 12 **c** 1, 3, 5, 7, 9
d -1, 1, 3, 5, 7 **e** 5, 7, 9, 11, 13 **f** 13, 15, 17, 19, 21
g 4, 7, 10, 13, 16 **h** 1, 5, 9, 13, 17

2 **a** 2, 4, 8, 16, 32 **b** 6, 12, 24, 48, 96
c 3, $1\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$ **d** -2, 4, -8, 16, -32

3 17, 11, 23, -1, 47

EXERCISE 2C

1 **a** $u_1 = 6$, $d = 11$ **b** $u_n = 11n - 5$ **c** 545
d yes, u_{30} **e** no

2 **a** $u_1 = 87$, $d = -4$, **b** $u_n = 91 - 4n$ **c** -69 **d** no

3 **b** $u_1 = 1$, $d = 3$ **c** 169 **d** $u_{151} = 451$

4 **b** $u_1 = 32$, $d = -\frac{7}{2}$ **c** -227 **d** $n \geqslant 68$

5 **a** $k = 17\frac{1}{2}$ **b** $k = 4$ **c** $k = 3$, $k = -1$

6 **a** $u_n = 6n - 1$ **b** $u_n = -\frac{3}{2}n + \frac{11}{2}$ **c** $u_n = -5n + 36$
d $u_n = -\frac{3}{2}n + \frac{1}{2}$

7 **a** $6\frac{1}{4}$, $7\frac{1}{2}$, $8\frac{3}{4}$ **b** $3\frac{5}{7}$, $8\frac{3}{7}$, $13\frac{1}{7}$, $17\frac{6}{7}$, $22\frac{4}{7}$, $27\frac{2}{7}$

8 **a** $u_1 = 36$, $d = -\frac{2}{3}$ **b** 100 **9** 100 006

EXERCISE 2D

1 **a** $b = 18$, $c = 54$ **b** $b = 2\frac{1}{2}$, $c = 1\frac{1}{4}$ **c** $b = 3$, $c = -1\frac{1}{2}$

2 **a** $u_1 = 5$, $r = 2$ **b** $u_n = 5 \times 2^{n-1}$, $u_{15} = 81\,920$

3 **a** $u_1 = 12$, $r = -\frac{1}{2}$ **b** $u_n = 12 \times (-\frac{1}{2})^{n-1}$, $u_{13} = \frac{3}{1024}$

4 **a** $u_1 = 8$, $r = -\frac{3}{4}$, $u_{10} = -0.600\,677\,49$

5 **a** $u_1 = 8$, $r = \frac{1}{\sqrt{2}}$, $u_n = 2^{\frac{7}{2} - \frac{n}{2}}$

6 **a** $k = \pm 14$ **b** $k = 2$ **c** $k = -2$ or 4

7 **a** $u_n = 3 \times 2^{n-1}$ **b** $u_n = 32 \times (-\frac{1}{2})^{n-1}$
c $u_n = 3 \times (\sqrt{2})^{n-1}$ **d** $u_n = 10 \times (\sqrt{2})^{1-n}$

8 **a** $u_9 = 13\,122$ **b** $u_{14} = 2916\sqrt{3} \doteqdot 5050.66$
c $u_{18} \doteqdot 0.000\,091\,55$ **9** **a** \$3993.00 **b** \$993.00

10 11 470.39 Euro **11** **a** 43 923 Yen **b** 13 923 Yen

12 \$23 602.32 **13** 148 024.43 Yen **14** £51 249.06

15 \$14 976.01 **16** £11 477.02 **17** 19 712.33 Euro

18 19 522.47 Yen

19 **a** **i** 1550 ants **ii** 4820 ants **b** 12.2 weeks

20 **a** 278 animals **b** Year 2037

EXERCISE 2E.1

1 **a** **i** $S_n = 3 + 11 + 19 + 27 + ... + (8n - 5)$ **ii** 95
b **i** $S_n = 42 + 37 + 32 + ... + (47 - 5n)$ **ii** 160
c **i** $S_n = 12 + 6 + 3 + 1\frac{1}{2} + ... + 12(\frac{1}{2})^{n-1}$ **ii** $23\frac{1}{4}$
d **i** $S_n = 2 + 3 + 4\frac{1}{2} + 6\frac{3}{4} + ... + 2(\frac{3}{2})^{n-1}$ **ii** $26\frac{3}{8}$
e **i** $S_n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^{n-1}}$ **ii** $1\frac{15}{16}$
f **i** $S_n = 1 + 8 + 27 + 64 + ... + n^3$ **ii** 225

EXERCISE 2E.2

1 **a** 820 **b** 3087.5 **c** -1460 **d** -740

2 **a** 1749 **b** 2115 **c** $1410\frac{1}{2}$ **3** 203

4 -115.5 **5** 18 **6** **a** 65 **b** 1914 **c** 47 850

7 **a** 14 025 **b** 71 071 **c** 3367

9 **a** $u_n = 2n - 1$ **c** $S_1 = 1$, $S_2 = 4$, $S_3 = 9$, $S_4 = 16$

10 56, 49 **11** 10, 4, -2 or -2, 4, 10

12 2, 5, 8, 11, 14 or 14, 11, 8, 5, 2

EXERCISE 2E.3

1 **a** $23.9766 \doteqdot 24.0$ **b** $\doteqdot 189\,134$ **c** $\doteqdot 4.000$ **d** $\doteqdot 0.5852$

2 **a** $S_n = \frac{3 + \sqrt{3}}{2}\left((\sqrt{3})^n - 1\right)$ **b** $S_n = 24(1 - (\frac{1}{2})^n)$
c $S_n = 1 - (0.1)^n$ **d** $S_n = \frac{40}{3}(1 - (-\frac{1}{2})^n)$

3 **c** \$26 361.59

4 **a** $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, $\frac{31}{32}$ **b** $S_n = \frac{2^n - 1}{2^n}$
c $1 - (\frac{1}{2})^n = \frac{2^n - 1}{2^n}$ **d** as $n \to \infty$, $S_n \to 1$

5 **b** $S_n = 1 + 18(1 - (0.9)^{n-1})$ **c** 19 seconds

6 **a** **i** $u_1 = \frac{3}{10}$ **ii** $r = 0.1$ **b** $S_\infty = \frac{1}{3}$

7 **a** $\frac{4}{9}$ **b** $\frac{16}{99}$ **c** $\frac{104}{333}$

EXERCISE 2F

1 **a** 10 **b** 25 **c** 168 **d** 310

2 $2 + 5 + 8 + 11 + ... + 59 = 610$

3 **a** 160 **b** -630 **c** 135

4 **a** 3069 **b** $\frac{4095}{1024} \doteqdot 3.999$ **c** $-134\,217\,732$

5 **a** 420 **b** 2231.868 211 **6** **a** $n = 37$ **b** $n = 11$

EXERCISE 2G

1 34th week (total sold = 2057)

2 After 85 months its value is \$501.88 and after 86 months its value is \$491.84, $\therefore$ during the 86th month its value is \$500.

3 54 or $\frac{2}{3}$ **4** 70 cm

5 The 20th terms are: arithmetic 39, geometric 3^{19}
or arithmetic $7\frac{1}{3}$, geometric $(\frac{4}{3})^{19}$

6 $x = \frac{1}{2}$ **7** **a** $u_1 = 7,\ u_2 = 10,$ **b** 64

8 **a** $A_3 = \$8000(1.03)^3 - (1.03)^2R - 1.03R - R$

b $A_8 = \$8000(1.03)^8 - (1.03)^7R - (1.03)^6R - (1.03)^5R - (1.03)^4R - (1.03)^3R - (1.03)^2R - (1.03)R - R = 0 \qquad R = \$1139.65$

REVIEW SET 2A

1 **a** $\frac{1}{3}$, 1, 3, 9 **b** $\frac{5}{4}, \frac{8}{5}, \frac{11}{6}$, 2 **c** 5, −5, 35, −65

2 **b** $u_1 = 63,\ d = -5$ **c** -117 **d** $u_{54} = -202$

3 **a** $u_1 = 3,\ r = 4$ **b** $u_n = 3 \times 4^{n-1},\ u_9 = 196\,608$

4 $k = -\frac{11}{2}$ **5** $u_n = 73 - 6n,\ u_{34} = -131$

6 **b** $u_1 = 6,\ r = \frac{1}{2}$ **c** 0.000 183

7 $u_n = 33 - 5n,\ S_n = \frac{n}{2}(61 - 5n)$ **8** $k = \pm\frac{2\sqrt{3}}{3}$

9 $u_n = \frac{1}{6} \times 2^{n-1}$ or $-\frac{1}{6} \times 2^{n-1}$

REVIEW SET 2B

1 **a** 81 **b** $-1\frac{1}{2}$ **c** −486 **2** 21, 19, 17, 15, 13, 11

3 **a** $u_n = 89 - 3n$ **b** $u_n = \dfrac{2n+1}{n+3}$ **c** $u_n = 100(0.9)^{n-1}$

4 **a** $1 + 4 + 9 + 16 + 25 + 36 + 49$

b $\frac{4}{3} + \frac{5}{4} + \frac{6}{5} + \frac{7}{6} + \frac{8}{7} + \frac{9}{8} + \frac{10}{9} + \frac{11}{10}$

5 **a** $\sum_{r=1}^{n}(7r-3)$ **b** $\sum_{r=1}^{n}(\frac{1}{2})^{r+1}$ **6** **a** 1587 **b** $47\frac{253}{256} \div 47.99$

7 **a** 70 **b** 241.2 **8** $u_{12} = 10\,240$

9 **a** 8415.31 Euro **b** 8488.67 Euro **c** 8505.75 Euro

REVIEW SET 2C

1 $u_n = (\frac{3}{4})2^{n-1}$ **a** 49 152 **b** 24 575.25 **2** 12

3 $u_{11} = \frac{8}{19\,683} \div 0.000\,406$ **4** **a** 17 **b** $255\frac{511}{512} \div 256.0$

5 **a** \$18 726.65 **b** \$18 885.74

6 \$13 972.28 **7** **a** 3470 **b** Year 2008

8 18 metres **9** **a** $0 < x < 1$ **b** $35\frac{5}{7}$

EXERCISE 3A

1 **a** $2^1 = 2,\ 2^2 = 4,\ 2^3 = 8,\ 2^4 = 16,\ 2^5 = 32,\ 2^6 = 64$

b $3^1 = 3,\ 3^2 = 9,\ 3^3 = 27,\ 3^4 = 81$

c $5^1 = 5,\ 5^2 = 25,\ 5^3 = 125,\ 5^4 = 625$

d $7^1 = 7,\ 7^2 = 49,\ 7^3 = 343$

EXERCISE 3B

1 **a** −1 **b** 1 **c** 1 **d** −1 **e** 1 **f** −1 **g** −1 **h** −8 **i** −8 **j** 8 **k** −25 **l** 125

2 **a** 512 **b** −3125 **c** −243 **d** 16 807 **e** 512 **f** 6561 **g** −6561 **h** 5.117 264 691 **i** −0.764 479 956 **j** −20.361 584 96

3 **a** $0.\overline{142\,857}$ **b** $0.\overline{142\,857}$ **c** $0.\overline{1}$ **d** $0.\overline{1}$ **e** 0.015 625 **f** 0.015 625 **g** 1 **h** 1 **4** 3 **5** 7

EXERCISE 3C

1 **a** 7^5 **b** 5^7 **c** a^9 **d** a^5 **e** b^{13} **f** a^{3+n} **g** b^{7+m} **h** m^9

2 **a** 5^7 **b** 11^4 **c** 7^3 **d** a^4 **e** b^3 **f** p^{5-m} **g** y^{a-5} **h** b^{2x-1}

3 **a** 3^8 **b** 5^{15} **c** 2^{28} **d** a^{10} **e** p^{20} **f** b^{5n} **g** x^{3y} **h** a^{10x}

4 **a** 2^3 **b** 5^2 **c** 3^3 **d** 2^6 **e** 3^4 **f** 3^{a+2} **g** 5^{t-1} **h** 3^{3n} **i** 2^{4-x} **j** 3^2 **k** 5^{4x-4} **l** 2^2 **m** 2^{y-2x} **n** 2^{2y-3x} **o** 3^{2x} **p** 2^3

5 **a** a^3b^3 **b** a^4c^4 **c** b^5c^5 **d** $a^3b^3c^3$ **e** $16a^4$ **f** $25b^2$ **g** $81n^4$ **h** $8b^3c^3$ **i** $64a^3b^3$ **j** $\dfrac{a^3}{b^3}$ **k** $\dfrac{m^4}{n^4}$ **l** $\dfrac{32c^5}{d^5}$

6 **a** $8b^{12}$ **b** $\dfrac{9}{x^4y^2}$ **c** $25a^8b^2$ **d** $\dfrac{m^{12}}{16n^8}$ **e** $\dfrac{27a^9}{b^{15}}$ **f** $32m^{15}n^{10}$ **g** $\dfrac{16a^8}{b^4}$ **h** $125x^6y^9$ **i** $4a^2$ **j** $36b^4$ **k** $-8a^3$ **l** $-27m^6n^6$ **m** $16a^4b^{16}$ **n** $\dfrac{-8a^6}{b^6}$ **o** $\dfrac{16a^6}{b^2}$ **p** $\dfrac{9p^4}{q^6}$

7 **a** a^2 **b** $8b^5$ **c** m^3n **d** $7a^5$ **e** $4ab^2$ **f** $\dfrac{9m^3}{2}$ **g** $40h^5k^3$ **h** $\dfrac{1}{m^5}$ **i** p^3

8 **a** 1 **b** $\frac{1}{3}$ **c** $\frac{1}{6}$ **d** 1 **e** 4 **f** $\frac{1}{4}$ **g** 8 **h** $\frac{1}{8}$ **i** 25 **j** $\frac{1}{25}$ **k** 100 **l** $\frac{1}{100}$

9 **a** 1 **b** 1 **c** 3 **d** 1 **e** 2 **f** 1 **g** $\frac{1}{25}$ **h** $\frac{1}{32}$ **i** 3 **j** $\frac{5}{2}$ **k** $\frac{3}{4}$ **l** 12 **m** $2\frac{1}{4}$ **n** $\frac{4}{5}$ **o** $\frac{8}{7}$ **p** $\frac{7}{2}$

10 **a** $\dfrac{1}{2a}$ **b** $\dfrac{2}{a}$ **c** $\dfrac{3}{b}$ **d** $\dfrac{1}{3b}$ **e** $\dfrac{b^2}{4}$ **f** $\dfrac{1}{4b^2}$ **g** $\dfrac{1}{9n^2}$ **h** $\dfrac{n^2}{3}$ **i** $\dfrac{a}{b}$ **j** $\dfrac{1}{ab}$ **k** $\dfrac{a}{b^2}$ **l** $\dfrac{1}{a^2b^2}$ **m** $\dfrac{1}{2ab}$ **n** $\dfrac{2}{ab}$ **o** $\dfrac{2a}{b}$ **p** a^2b^3

11 **a** 3^{-1} **b** 2^{-1} **c** 5^{-1} **d** 2^{-2} **e** 3^{-3} **f** 5^{-2} **g** 2^{-3x} **h** 2^{-4y} **i** 3^{-4a} **j** 3^{-2} **k** 5^{-2} **l** 5^{-3} **m** 2^4 **n** $2^0 = 3^0 = 5^0$ **o** $2^{-3} \times 3^{-3}$ **p** $2^4 \times 5^2$

12 25 days **13** **a** $5^3 = 21 + 23 + 25 + 27 + 29$

b $7^3 = 43 + 45 + 47 + 49 + 51 + 53 + 55$

c $12^3 = 133 + 135 + 137 + 139 + 141 + 143 + 145 + 147 + 149 + 151 + 153 + 155$

14 5^{75}

EXERCISE 3D

1 **a** $2^{\frac{1}{5}}$ **b** $2^{-\frac{1}{5}}$ **c** $2^{\frac{3}{2}}$ **d** $2^{\frac{5}{2}}$ **e** $2^{-\frac{1}{3}}$ **f** $2^{\frac{4}{3}}$ **g** $2^{\frac{3}{2}}$ **h** $2^{\frac{3}{2}}$ **i** $2^{-\frac{4}{3}}$ **j** $2^{-\frac{3}{2}}$

2 **a** $3^{\frac{1}{3}}$ **b** $3^{-\frac{1}{3}}$ **c** $3^{\frac{1}{4}}$ **d** $3^{\frac{3}{2}}$ **e** $3^{-\frac{5}{2}}$

3 **a** $7^{\frac{1}{3}}$ **b** $3^{\frac{3}{4}}$ **c** $2^{\frac{4}{5}}$ **d** $2^{\frac{5}{3}}$ **e** $7^{\frac{2}{7}}$ **f** $7^{-\frac{1}{3}}$ **g** $3^{-\frac{3}{4}}$ **h** $2^{-\frac{4}{5}}$ **i** $2^{-\frac{5}{3}}$ **j** $7^{-\frac{2}{7}}$

4 **a** 2.280 **b** 1.834 **c** 0.794 **d** 0.435

5 **a** 3 **b** 1.682 **c** 1.933 **d** 0.523

6 **a** 8 **b** 32 **c** 8 **d** 125 **e** 4 **f** $\frac{1}{2}$ **g** $\frac{1}{27}$ **h** $\frac{1}{16}$ **i** $\frac{1}{81}$ **j** $\frac{1}{25}$

EXERCISE 3E

1 **a** $x^5 + 2x^4 + x^2$ **b** $2^{2x} + 2^x$ **c** $x + 1$ **d** $e^{2x} + 2e^x$ **e** $2 \times 3^x - 1$ **f** $x^2 + 2x + 3$ **g** $1 + 5 \times 2^{-x}$ **h** $5^x + 1$ **i** $x^{\frac{3}{2}} + x^{\frac{1}{2}} + 1$

2 **a** $4^x + 2^{2+x} + 3$ **b** $9^x + 7 \times 3^x + 10$ **c** $25^x - 6 \times 5^x + 8$ **d** $4^x + 6 \times 2^x + 9$ **e** $9^x - 2 \times 3^x + 1$ **f** $16^x + 14 \times 4^x + 49$ **g** $x - 4$ **h** $4^x - 9$ **i** $x - x^{-1}$ **j** $x^2 + 4 + \dfrac{4}{x^2}$ **k** $e^{2x} - 2 + e^{-2x}$ **l** $25 - 10 \times 2^{-x} + 4^{-x}$