

FINDING THE TIME PERIOD

Often we wish to know **how long** we must invest money for it to grow to a specified sum in the future.

Example 20

For how long must Magnus invest \$4000 at 6.45% p.a. compounded half-yearly if it is to amount to \$10 000?

Formula solution:

Given $A = 10\,000$

$C = 4000$

$$r = \frac{6.45}{2} = 3.225$$

Using $A = C \times \left(1 + \frac{r}{100}\right)^n$

$$\therefore 10\,000 = 4000 \times \left(1 + \frac{3.225}{100}\right)^n$$

$$\therefore 10\,000 = 4000 \times (1.03225)^n$$

$$\therefore n = 28.9 \quad \{\text{using solver}\}$$

Thus, 29 half-years are required, i.e., 14.5 years.

Graphics calculator:

To answer this using the TVM function on the calculator, set up the TVM screen as shown. We then need to find the number of periods required.

For TI-83:

```

N=28.86783747
I%=6.45
PV=-4000
PMT=0
FV=10000
P/Y=2
C/Y=2
PMT: [END] BEGIN
  
```

For Casio:

```

Compound Interest:End
n = 28.86783747
I% = 6.45
PV = -4000
PMT = 0
FV = 10000
P/Y = 2
↓
| n | I% | PV | PMT | FV | AMT |
  
```

$n = 28.9$, thus, 29 half-years are required, i.e., 14.5 years.

Note: We find the number of compounding periods and need to convert to the time units required. The appropriate time periods in the above example are years.

EXERCISE 13C.6

- 1 Your parents give you \$8000 to buy a car but the car you want costs \$9200. You deposit this money in an account that pays 6% p.a. compounded monthly. How long will it be before you have enough money to buy the car you want?
- 2 A couple inherit \$40 000 and deposit it in an account that pays $4\frac{1}{2}\%$ p.a. compounded quarterly. They withdraw the money as soon as they have over \$45 000. How long did they keep the money in that account?
- 3 A business deposits £80 000 in an account that pays $5\frac{1}{4}\%$ p.a. compounded monthly. How long will it take before they double their money?
- 4 An investor deposits \$12 000 in an account paying 5% p.a. compounded daily. How long will it take the investor to earn \$5000 in interest?

- 2 a i \$7.20 US ii \div £210 b i \$12.60 US ii \div AUD \$944
 c i \$23.40 US ii \div 1027 euro
 3 a \div 1392 baht b \div 388 pesos c \div 12 pesos
 4 a \div 2837 rupees b \div 363 rand c \div 12 rand

EXERCISE 13A.4

- 1 a \$2486.80 US b 2813.20 Norwegian kroner
 c \$18 759.95 NZ d 22 994.60 Thai baht e 110 830.85 Yen

EXERCISE 13B.1

- 1 a \$480 b \$1239.75 c \$6933.33 d 402.07 Yen
 2 \$15 000 at 7% p.a. for 4 years is less by \$221.25
 3 a \$2800 b £21 000 c \$124 444.45
 5 a $3\frac{1}{3}\%$ p.a. b 8% p.a. c \div 7.09% p.a.
 7 20% p.a. 8 a $2\frac{1}{2}$ years b $6\frac{1}{2}$ years c 6 years

EXERCISE 13B.2

- 1 \$274.84 2 787.50 baht 3 \$1418.75 4 \$1660
 5 Accept the loan from Lesley (\$269.73 per month) compared to paying Rachel \$338.22 per month.

EXERCISE 13C.1

- 1 a \$5359.57 b \$7293.04 c £9300.65
 2 a 113.40 euro b \$1170.25 c \$6663.24
 3 a \$17 496 b \$2496
 4 a 9663.60 kroner b 1663.60 kroner

EXERCISE 13C.2

- 1 \$9021.59 2 \$301.25
 3 a \$7650 b \$8151.65 c \$8243.80
 Clearly, compounding quarterly is \$92.15 more than compounding half-yearly and \$593.80 more than simple interest.

EXERCISE 13C.3

- 1 £13 373.53 2 \$546.01
 3 a 4725 Yen b 4940.81 Yen c 4998.89 Yen
 d 5028.61 Yen
 e 5048.67 Yen - Clearly, interest compounded monthly gives the greatest return.
 4 Bank A balance = \$157 149.29, Bank B balance = \$155 344.23
 Jai should invest with Bank A.
 5 Simple interest earned = 8960 euro
 Interest compounded monthly = 9760.78 euro
 Interest compounded monthly is 800.78 euro more than simple interest.

EXERCISE 13C.4

- 1 \$6631 2 \$4079.75 3 \$4159.10
 4 a 308.70 rupees interest b 636.36 rupees interest
 c 1351.20 rupees interest Doubling the interest rate more than doubles the interest earned.

EXERCISE 13C.5

- 1 19 971.30 Yen 2 \$20 836.90 3 \$80 000
 4 80 162.65 baht 5 \$1564.95

EXERCISE 13C.6

- 1 \div 28 months (2 years, 4 months) 2 11 quarters (2.75 years)
 3 159 months (13 years, 3 months)
 4 2543 days (6 years, 353 days)

EXERCISE 13C.7

- 1 14.47% p.a. 2 6.0% p.a. 3 5.25% p.a.
 4 Shares 13.05% p.a., house 9.48% p.a., i.e., the shares had the greater average annual percentage increase in value.

EXERCISE 13C.8

- 1 a \$1676.03 b \$1669.09 \therefore a is more by \$6.94
 2 a Monthly: \$863.16, half-yearly: \$859.58
 b Monthly: \$1148.08, half-yearly: \$1143.33
 3 a Derk should choose the 6.40% interest paid at maturity (i.e., annually).
 b \$3200 c \$1808

EXERCISE 13C.9

- 1 A is effectively 5.47% p.a., B is effectively 5.41% p.a., \therefore A is the better rate.
 2 P is effectively 7.87% p.a., Q is effectively 7.90% p.a., \therefore Q is the better rate.
 3 a 4.95% p.a. b 5.01% p.a.
 c 4.9% p.a. compounded monthly
 4 a 8.06% p.a. b 8.11% p.a.
 c 7.95% p.a. compounded half-yearly
 5 12 times (monthly)
 6 a 6.379% p.a. b 6.398% p.a. c 6.399% p.a. d 6.40% p.a.
 e Take interest at maturity to receive \$2560 interest.

EXERCISE 13C.10

- 1 a 6.58% p.a. b 5.76% p.a. c 5.15% p.a. d 4.69% p.a.
 2 a 7.51% p.a. b 4.30% p.a. c 4.97% p.a. d 3.62% p.a.
 3 6.38% p.a.

EXERCISE 13D

1 a

Age (years)	Depreciation	Book value
0	\$0	\$15 000
1	\$2250	\$12 750
2	\$1912.50	\$10 837.50
3	\$1625.63	\$9211.87

- b i \$2250 ii \$1912.50 iii \$1625.63
 2 a \$53 393.55 b \$171 606.45
 3 a \$16 588.80 b \$15 811.20 c \div 24.79% p.a.
 5 a \div 18.37% p.a. 6 a \$175 702.59 b \$64 297.41

EXERCISE 13E

- 1 a \$267 b \$16 020 c \$4020 d \$11 704.53
 2 a \$308.80 b \$11 116.80 c \$1616.80 d \$8817.04
 3 a 2032.80 Yen b 5250 Yen
 4 a \$7994 b \$5110.40
 Becky should borrow from the Cash Credit Union as she pays \$2883.60 less in interest (although her monthly repayments are more.)

EXERCISE 13F.1

- 1 a \$66 354.49 b \$56 020.65 c \$10 333.84 d 3.44% p.a.
 2 a \$22 444.54 b \$22 110.15 c \$334.39 d 0.50% p.a.
 3 Decreased by \$20.49
 4 a \$1343.92 b \$1806.11 c \$2427.26

EXERCISE 13F.2

- 1 a \$18 013.95 b \$15 876.55
 2 a \$12 916.22 b i \$10 090.13 ii \$9156.55 iii \$8317.11
 3 a \$7812.50 b i \$21 049.33 ii \$19 128.36
 4 a \$3060.45 b \$2596.06
 c Jon needed to invest at greater than the inflation rate (i.e., 4.2% p.a.).

REVIEW SET 13A

- 1 \$1199.90 US 2 248.60 euro 3 \div 5.43 years