Exercises 1.2

1. a. If x has coordinates (u, v), what coordinates does $-x$ have?
   b. $\cos x = u$. What is $\cos(-x)$?
   c. $\sin x = v$. What is $\sin(-x)$?

2. Find the following function values.

   a. $\cos\left[-\frac{\pi}{3}\right]$ b. $\cos\left[-\frac{\pi}{4}\right]$ c. $\cos\left[-\frac{\pi}{6}\right]$

   d. $\sin\left[-\frac{\pi}{3}\right]$ e. $\sin\left[-\frac{\pi}{4}\right]$ f. $\sin\left[-\frac{\pi}{6}\right]$

3. a. If x has coordinates (u, v), what coordinates does $x + \pi$ have?
   b. What is $\cos(x + \pi)$?
   c. What is $\sin(x + \pi)$?

4. Find the following function values.

   a. $\cos\frac{7\pi}{6}$ b. $\cos\frac{5\pi}{4}$ c. $\cos\frac{4\pi}{3}$

   d. $\sin\frac{7\pi}{6}$ e. $\sin\frac{5\pi}{4}$ f. $\sin\frac{4\pi}{3}$

5. a. If x has coordinates (u, v), what coordinates does $\pi - x$ have?
   b. What is $\cos(\pi - x)$?
   c. What is $\sin(\pi - x)$?

6. Find the following function values.

   a. $\cos\frac{3\pi}{4}$ b. $\cos\frac{2\pi}{3}$ c. $\cos\frac{5\pi}{6}$

   d. $\sin\frac{3\pi}{4}$ e. $\sin\frac{2\pi}{3}$ f. $\sin\frac{5\pi}{6}$

7. If x has coordinates (u, v), what coordinates does $x \pm 2\pi$ have?

   a. What is $\cos(x + 2\pi)$? b. What is $\sin(x + 2\pi)$?
   c. What is $\cos(x - 2\pi)$? d. What is $\sin(x - 2\pi)$?

8. Use the results of exercises 1–7 to find the following function values

   a. $\cos\left[\frac{7\pi}{4}\right]$ b. $\cos\left[-\frac{7\pi}{6}\right]$ c. $\cos\left[\frac{10\pi}{3}\right]$

   d. $\sin\left[-\frac{5\pi}{4}\right]$ e. $\sin\left[\frac{11\pi}{6}\right]$ f. $\sin\left[-\frac{4\pi}{3}\right]$

   g. $\sin\left[-\frac{5\pi}{6}\right]$ h. $\sin\left[\frac{8\pi}{3}\right]$ i. $\sin\left[-\frac{9\pi}{4}\right]$

   j. $\cos\left[\frac{5\pi}{6}\right]$ k. $\cos\left[-\frac{11\pi}{6}\right]$ l. $\cos\left[\frac{13\pi}{4}\right]$

Use the fact that $\sin^2 x + \cos^2 x = 1$ and the given quadrant to find the function value indicated.

Example

x is in the third quadrant, $\sin x = -\frac{12}{13}$. Find $\cos x$.

Solution

Since x is in the third quadrant, $\cos x$ is negative.

$$\sin^2 x + \cos^2 x = 1 \qquad \left[\frac{12}{13}\right]^2 + \cos^2 x = 1$$

$$\frac{144}{169} + \cos^2 x = 1 \qquad \cos^2 x = \frac{25}{169}$$

$$\cos^2 x = 1 - \frac{144}{169} \qquad \cos x = -\frac{5}{13} \text{ (since } \cos x \text{ is negative}$$

9. x is in the second quadrant, $\cos x = -\frac{15}{17}$. Find $\sin x$.
10. x is in the fourth quadrant, $\cos x = \frac{3}{5}$. Find $\sin x$.
11. x is in the fourth quadrant, $\sin x = -\frac{1}{3}$. Find $\cos x$.
12. x is in the second quadrant, $\sin x = \frac{\sqrt{5}}{5}$. Find $\cos x$.
13. z is in the first quadrant, $\sin z = \frac{8}{17}$. Find $\cos z$.
14. z is in the third quadrant, $\sin z = -\frac{3}{4}$. Find $\cos z$.
15. y is in the third quadrant, $\cos y = -\frac{2}{5}$. Find $\sin y$.
16. t is in the first quadrant, $\cos t = \frac{5}{13}$. Find $\sin t$.
17. w is in the fourth quadrant, $\sin w = -\frac{2}{3}$. Find $\cos w$.
18. θ is in the second quadrant, $\cos \theta = -\frac{2}{7}$. Find $\sin \theta$.

Answers

Exercises 1.2

1. a. $(u, -v)$ b. u c. $-v$
2. a. $\frac{1}{2}$ c. $\frac{\sqrt{3}}{2}$ e. $-\frac{\sqrt{2}}{2}$
3. a. $(-u, -v)$ b. $-u$ c. $-v$
4. a. $-\frac{\sqrt{3}}{2}$ c. $-\frac{1}{2}$ e. $-\frac{\sqrt{2}}{2}$
5. a. $(-u, v)$ b. $-u$ c. v
6. a. $-\frac{\sqrt{2}}{2}$ c. $-\frac{\sqrt{3}}{2}$ e. $\frac{\sqrt{3}}{2}$

9. $\frac{8}{17}$ 11. $\frac{2\sqrt{2}}{3}$ 13. $\frac{15}{17}$ 15. $-\frac{\sqrt{21}}{5}$ 17. $\frac{\sqrt{5}}{3}$