

Exercises ^[B]

1. Before logarithms were invented, tables giving the "quarter squares" of numbers were used in multiplication. The quarter square of x is $\frac{1}{4}x^2$. The rule for obtaining the product of numbers a and b was: "Subtract the quarter square of the difference of a and b from the quarter square of the sum of a and b ; the result is the product ab ." Demonstrate the validity of the rule.
2. Simplify $\log(x + \sqrt{x^2 - 1}) + \log(x - \sqrt{x^2 - 1})$.
3. If $XY = 10^{2.73}$, find the value of $\log_{10}X + \log_{10}Y$.
4. If $4^x = 10$, show by taking logarithms of both sides that $x = \frac{1}{\log_{10}4}$, and so find the value of x to 4 significant figures.
5. Evaluate $\log_5 30$. (This means: Find x where $5^x = 30$.)
6. If $x = \log_b N$, then $b^x = N$. By taking logarithms to base 10 of both sides of the latter statement, show that $\log_b N = \frac{\log_{10} N}{\log_{10} b}$.
7. Find x if $10^x = 0.25$.
8. To evaluate $\log_2 3$ a student argued that $\log_2 2 = 1$ and $\log_2 4 = 2$, so that $\log_2 3 = 1.5$ by interpolation. Why is this not valid? Find the required value to 4 significant figures.
9. Find the value of x for which $(0.55)^x = 2.718$.
10. In combining $\log 1 + \log 2 + \log 3$, a student argued that

$$\log 1 + \log 2 + \log 3 = \log(1 + 2 + 3) = \log 6.$$
 Is his procedure correct? Is the result correct? If it is required that $\log(a + b + c) = \log a + \log b + \log c$, express c in terms of a and b .
11. If $\log x = n[\log(n + 1) - \log n]$, express x in terms of n .
12. Using the approximations $e = 2.718$, $\pi = 3.142$, evaluate (a) e^3 ; (b) $e^{3.2}$; (c) e^π .
13. Express X in terms of A , e , and t , if $\log_e X - \log_e A = 2t$.

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| <u>2.</u> 0 | <u>8.</u> $\log_2 3 = 1.585$ | <u>11.</u> $x = (1 + \frac{1}{n})^n$ |
| <u>3.</u> 2.73 | <u>9.</u> $x = -1.673$ | <u>12.</u> a. 20.09 |
| <u>4.</u> $x = 1.661$ | <u>10.</u> No; yes; | b. 24.53 |
| <u>5.</u> 2.113 | $c = \frac{a + b}{ab - 1}$ | c. 23.17 |
| <u>7.</u> $x = -0.6021$ | | <u>13.</u> $X = A \cdot e^{2t}$ |