In the preceding example the base of the logarithms was omitted because transformations made by means of the laws of logarithms are valid for all usable bases. In this section of our work the base 10 has no particular importance, and when it is necessary to use 10 as a base, it will be written as it was in the first example. The absence of a base in the following exercises is an indication that the required result is valid for any base.

Exercises [A]

In exercises 1–10, use the laws of logarithms to reduce each expression to the form $\log n$.

1. $\log 2 + \log 4$
2. $\log 15 - \log 5$
3. $\log 4 + \log \frac{1}{4}$
4. $\log 5\frac{1}{3} + \log \frac{3}{4}$
5. $\log 1\frac{1}{4} - \log \frac{1}{8}$
6. $2 \log 6 + \log \frac{1}{4} - \log 3$
7. $\frac{1}{2} \log 16 + \log 5 + \log \frac{1}{2}$
8. $2 \log 4 - \frac{1}{3} \log 8$
9. $\log 2x - \log x$
10. $\log \sqrt{x} + \frac{1}{2} \log \frac{1}{x}$

In exercises 11–20, express each formula as a relationship between the logarithms of the individual letters and numbers involved, to the extent that this is possible.

Example. If $V = \frac{4}{3} \pi R^3$, $\log V = \log 4 + \log \pi + 3 \log R - \log 3$.

11. $C = \pi D$
12. $K = \frac{1}{2} bh$
13. $A = \pi R^2$
14. $T = ar^5$
15. $h = \dfrac{3V}{e^2}$
16. $t = 2\pi \sqrt{\dfrac{L}{g}}$
17. $V = \dfrac{S^{\frac{3}{2}}}{6\pi^{\frac{1}{2}}}$
18. $A = P(1+r)^n$
19. $x = \sqrt{a^2 - ab}$
20. $S = 2\pi rh + 2\pi r^2$
21. Evaluate $\log_2 12 + \log_2 1\frac{1}{3}$.
22. Evaluate $3 \log_{10} 5 + \log_{10}(0.8)$.
23. Evaluate $\log_2 \sqrt{28} - \frac{1}{2} \log_2 3.5$.
24. Evaluate $2 \log_8 12 - \frac{1}{2} \log_8 81$.
25. Find x in terms of M and N, if
    (a) $\log x = \log(M+N) + \log(M-N)$.
    (b) $\log x = (\log M + \log N) + (\log M - \log N)$.
26. Show that $\log(\frac{1}{3} + \frac{1}{5}) - (\log \frac{1}{3} + \log \frac{1}{5}) = 3 \log 2$.
27. Show that $\log\left(\dfrac{1}{a} + \dfrac{1}{b}\right) - \left(\log \dfrac{1}{a} + \log \dfrac{1}{b}\right) = \log(a+b)$.
28. If $\log_{10} y = 1 + \log_{10} x$, express y in terms of x.
29. If $\log a - \log b = \log b - \log c$, express b in terms of a and c.
30. If $\log y = \log 3 + 2 \log x$, express y in terms of x.

1. $\log 8$

2. $\log 3$

3. $\log 1 = 0$

4. $\log 4$

5. $\log 10$

6. $\log 3$

7. $\log 10$

8. $\log 8$

9. $\log 2$

10. 0

11. $\log C = \log \pi + \log D$

12. $\log K = \log \frac{1}{2} + \log b + \log h$

13. $\log A = \log \pi + 2 \log R$

14. $\log T = \log a + 5 \log r$

15. $\log h = \log 3 + \log V - 2 \log e$

16. $\log t = \log 2 + \log \pi + \frac{1}{2}(\log L - \log g)$

17. $\log V = \frac{3}{2} \log S - (\log 6 + \frac{1}{2} \log \pi)$

18. $\log A = \log P + n \log (1 + r)$

19. $\log x = \frac{1}{2}[\log a + \log (a - b)]$

20. $\log S = \log 2 + \log \pi + \log r + \log (h + r)$

21. 4

22. 2

23. $\frac{3}{2}$

24. $\frac{4}{3}$

25. a. $x = M^2 - N^2$

    b. $x = M^2$

28. $y = 10x$

29. $b = \sqrt{ac}$

30. $y = 3x^2$