Exercises [B]

1. If $A = \frac{a^x + a^{-x}}{2}$, and $B = \frac{a^x - a^{-x}}{2}$, show that $A^2 - B^2 = 1$.

2. Simplify, by expressing each factor as a power of 2:

$$2^{-n} \cdot 8^{n-1} \cdot 4^{n+3} \div 16^n.$$

3. (a) Divide 10^n by 10. (b) Divide 10^n by 100. (c) Simplify $\left[\frac{10^{n+2}}{100}\right]^{\frac{1}{n}}$.

4. If $x^{\frac{1}{3}} y^{\frac{3}{4}} z^{-\frac{1}{6}} = 3$, express z as a power of 3 when both x and y have value 3.

5. (a) Multiply 2^x by 2. (b) Show that $2^{x+2} - (2^{x+1} + 2^x) = 2^x$.

6. (a) Show that $\frac{(x+1)^{\frac{1}{2}} + (x+1)^{-\frac{1}{2}}}{(x+1)^{\frac{1}{2}} - (x+1)^{-\frac{1}{2}}}$ reduces to $\frac{x+2}{x}$ for all positive values of x.

   (b) Show that $\frac{(x+1)^{\frac{1}{2}} + (x-1)^{\frac{1}{2}}}{(x+1)^{\frac{1}{2}} - (x-1)^{\frac{1}{2}}}$ reduces to $x + (x^2-1)^{\frac{1}{2}}$ for all values of x greater than or equal to 1.

7. Show that there are two rational numbers which satisfy the equation $x^{\frac{2}{3}} - 3x^{\frac{1}{3}} = 10$.

8. Show that there are four real, but no rational, numbers which satisfy the equation $x^2 + 10x^{-2} = 7$.

9. If $x = \frac{1}{2}(y + y^{-1})$, show that $y = x \pm (x^2 - 1)^{\frac{1}{2}}$, and that the two values of y are reciprocal quantities.

10. If $x = \frac{3}{t^{\frac{2}{3}} + t^{-\frac{1}{3}}}$, and $y = \frac{3}{t^{\frac{1}{3}} + t^{-\frac{2}{3}}}$, show that:
    (a) $y^3 = tx^3$; (b) $x^3 + y^3 = 3xy$.

Simplify:

11. $(x-1)^{\frac{1}{2}} + x(x-1)^{-\frac{1}{2}}$.
12. $(x^2+2)^{\frac{1}{2}} - x^2(x^2+2)^{-\frac{1}{2}}$.
13. $(2x+1)^{\frac{2}{3}} - 4(2x+1)^{-\frac{1}{3}}$.
14. $(1-x^2)^{-\frac{3}{2}} - 3x^2(1-x^2)^{-\frac{5}{2}}$.
15. $\frac{(a^2-x^2)^{\frac{1}{2}} + x^2(a^2-x^2)^{-\frac{1}{2}}}{a^2-x^2}$.
16. $\frac{(x+3)^{\frac{1}{4}}}{x} \cdot \left[\frac{(x+3)^{\frac{3}{4}} - x(x+3)^{-\frac{1}{4}}}{x+3}\right]$.

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2. 8

3. a. 10^{n-1}

b. 10^{n-2}

c. 10

4. $z = 3^{\frac{1}{2}}$

5. a. 2^{x+1}

7. {125, -8}

8. $\{\sqrt{5}, -\sqrt{5}, \sqrt{2}, -\sqrt{2}\}$

9. $y_1 = x + \sqrt{x^2 - 1}$

$y_2 = x - \sqrt{x^2 - 1}$

11. $\dfrac{2x - 1}{(x - 1)^{\frac{1}{2}}}$

12. $\dfrac{2}{(x^2 + 2)^{\frac{1}{2}}}$

13. $\dfrac{2x - 3}{(2x + 1)^{\frac{1}{3}}}$

14. $\dfrac{1 - 4x^2}{(1 - x^2)^{\frac{5}{2}}}$

15. $\dfrac{a}{(a^2 - x^2)^{\frac{3}{2}}}$

16. $\dfrac{3}{x(x + 3)}$